This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

NYPL RESEARCH LIBRARIES
3 3433 06272683 5

Fish

Robinson's Shorter Course.

THE COMPLETE ARITHMETIC.

ORAL AND WRITTEN.

SECOND PART.

BY DANIEL W. FISH, A.M.,

AUTHOR OF ROBINSON'S SERIES OF PROGRESSIVE ARITHMETICS

IVISON, BLAKEMAN, TAYLOR & CO.,
NEW YORK AND CHICAGO.
1875.

THE NEW YORK
PUBLIC LIBRARY
915545
ASTOR, LENOX AND
TILDEN FOUNDATIONS
R 1920 L

ROBINSON'S
SHORTER COURSE.

FIRST BOOK IN ARITHMETIC. PRIMARY.
COMPLETE ARITHMETIC. *In One volume.**
COMPLETE ALGEBRA.

ARITHMETICAL PROBLEMS. ORAL and WRITTEN.
ALGEBRAIC PROBLEMS.

KEYS to COMPLETE ARITHMETIC *and* PROBLEMS, *and*
to COMPLETE ALGEBRA *and* PROBLEMS,
in separate volumes, for Teachers.

Arithmetic, ORAL *and* WRITTEN, *usually taught in* THREE *books, is now offered, complete and thorough, in* ONE *book,* "THE COMPLETE ARITHMETIC."

---

* *This* COMPLETE ARITHMETIC *is also published in* TWO VOLUMES. ***PART I.*** *and* ***PART II.*** *are each bound separately, and in* CLOTH.

Copyright, 1874, by DANIEL W. FISH.

Electrotyped by SMITH & MCDOUGAL, 82 Beekman St., N. Y.

PREFACE

THE design of the author, in the preparation of the COMPLETE ARITHMETIC, has been to furnish a text-book on the subject of arithmetic, complete not only as a treatise, but as a comprehensive manual for the class-room, and, therefore, embodying every necessary form of illustration and exercise, both *oral* and *written*. Usually, this subject has been treated in such a way as to form the contents of *three or more graded text-books*, the oral exercises being placed in a separate volume. In the present treatise, however, the *whole subject* is presented in all its different grades; and the oral, or mental, arithmetic, so called, has been inserted, where it logically and properly belongs, either as introductory to the enunciation of principles or to the statement of practical rules—the treatment of every topic from the beginning to the end of the book being thoroughly *inductive*.

In this way, and by carefully constructed *analyses*, applied to all the various processes of mental arithmetic, the pupil's mind cannot fail to become thoroughly imbued with clear and accurate ideas in respect to each particular topic before he is required to learn, or apply to written examples, any set rule whatever. The intellect of the pupil is thus addressed at every step; and every part of the instruction is made the means of effecting that *mental development* which constitutes the highest aim, as well as the most important result, of every branch of education.

This mode of treatment has not only the advantage of logically training the pupil's mind, and cultivating his powers of calculation, but must also prove a *source* of *economy*, both of *time* and *money*, inasmuch as it is the means of substituting a single volume for an *entire series of text-books*.

As the time of many pupils will not permit them to pursue this study through all of its departments, the work is issued in *two parts*, as well as in a *single volume*. This will, it is thought, be also convenient for *graded schools*, in supplying a separate book for classes of the *higher* and *lower grades* respectively, without requiring any unnecessary repetition or review.

In this, the SECOND PART, all the higher departments of arithmetic including *Mensuration* are presented, commencing with *Percentage*, the study of which can be taken up by the pupil immediately on completing the FIRST PART. This part of the subject has been treated in a comprehensive manner, and is, in all respects, adapted to the wants of the present time, recognizing and explaining all the recent changes in Custom-house Business, Exchange, etc.

An Appendix of *forty-eight pages* of valuable Tables and Problems has been added to *this part* of the work, containing much useful and practical information, *fresh* and *important*, obtained by much labor of research and inquiry, which, with many other improvements, particularly adapt this work to the wants of the student qualifying for business, and of graduating classes in High Schools and Academies, as well as of Mercantile and Commercial colleges.

The *Reviews* interspersed throughout the book will be found to be just what is needed by the student to make his progress sure at each step, and to give him comprehensive ideas of the subject as he advances. Carefully constructed *Synopses* have also been inserted, with the view to afford to both teacher and pupil a ready means of *drill* and *examination*, as well as to present, in a clear, concise, and logical manner, the relations of all the different departments of the subject, with their respective *sub-topics*, *definitions*, *principles*, and *rules*.

It is confidently believed that, on examination, the work as a *whole*, as well as in its separate *parts*, will commend itself to teachers and others, by the careful *grading of its topics ;* the clearness and conciseness of its *definitions* and *rules ;* its improved methods of *analysis* and *operation ;* the great *number* and *variety* of its examples, both *oral* and *written*, embodying and elucidating all the *ordinary business* transactions ; and in the omission of all obsolete *terms* and *discarded usages*, as well as in the introduction of many *novel features* favorable both to *clearness* and *brevity*.

Great pains have also been taken to make this work superior to all others in its typographical arrangement and finish, and in the general tastefulness of its mechanical execution.

The author takes pleasure in acknowledging his indebtedness for many valuable suggestions received from teachers of experience and others interested in the work of education; especially to Joseph Ficklin, Ph. D., Professor of Mathematics in the University of Missouri, by whom chiefly the sections upon Involution, Evolution, Progressions, and Annuities have been prepared; as well as to Henry Kiddle, A. M., Superintendent of Schools in the city of New York, for valuable assistance, especially in the higher departments of Percentage, and for important suggestions in relation to other parts of the work.

D. W. F.

BROOKLYN, *January*, 1875.

SUGGESTIONS TO TEACHERS

IN order to teach any subject with the best success, the instructor should not only fully understand it, in all its principles and details, but should also clearly perceive what particular faculties of the mind are concerned in its acquisition and use.

Arithmetic is pre-eminently a subject of practical value; that is, it is one to be constantly applied to the practical affairs of life. But this is true only in a limited sense. Very few ever need to apply to any of the purposes of business more than a small part of the principles and rules of calculation taught in the text-books. Every branch of business has its own requirements in this respect, and these are all confined within very narrow limits.

The teaching of arithmetic must, therefore, to a great extent, be considered as *disciplinary*,—as training and developing certain faculties of the mind, and thus enabling it to perform its functions with accuracy and dispatch. The following *suggestions*, having reference to this twofold object of arithmetical instruction are presented to the teacher, as a partial guide, not only in the use of this text-book, but in the treatment of the subject as a branch of education.

Seek to cultivate in the pupil the habit of *self-reliance*. Avoid doing for him anything which, either with or without assistance, he should be able to do for himself. Encourage and stimulate his exertions, but do not supersede them.

Never permit him to accept any statement as true which he does not understand. Let him learn not by authority but by demonstration addressed to his own intelligence. Encourage him to ask questions and to interpose objections. Thus he will acquire that *most important of all* mental habits, that of *thinking for himself*.

Carefully discriminate, in the instruction and exercises, as to which faculty is addressed,—whether that of *analysis* or *reasoning*, or that of *calculation*. Each of these requires peculiar culture, and each has its appropriate period of development. In the *first stage* of arithmetical instruction, *calculation* should be chiefly addressed, and analysis or reasoning employed only after some progress has been made, and then very slowly and progressively. A young child will perform many operations in calculation which are far beyond its powers of analysis to explain thoroughly.

In the exercise of the calculating faculty, the examples should be rapidly performed, without pause for explanation or analysis ; and they should have very great variety, and be carefully arranged so as to advance from the simple and rudimental to the complicated and difficult.

In the exercise of the analytic faculty, great care should be taken that the processes do not degenerate into the mere repetition of *formulæ*. These forms of expression should be as simple and concise as possible, and should be, as far as practicable, expressed in the pupil's own language. Certain necessary points being attended to, the precise form of expression is of no more consequence than any particular letters or diagrams in the demonstration of geometrical theorems. Of course, the teacher should carefully criticise the logic or reasoning, not so as to discourage, but still insisting upon *perfect accuracy* from the first.

The oral or mental arithmetic should go hand in hand with the written. The pupil should be made to perceive that, except for the difficulty in retaining long processes in the mind, all arithmetic ought to be oral, and that the slate is only to be called into requisition to aid the mind in retaining intermediate processes and results. The arrangement of this text-book is particularly favorable for this purpose.

Definitions and principles should be carefully committed to memory. No slovenliness in this respect should be permitted. A definition is a basis for thought and reasoning, and every word which it contains is necessary to its integrity. A child should not be expected to frame a good definition. Of course, the pupil should be required to examine and criticise the definitions given, since this will conduce to a better understanding of their full meaning.

In conducting recitations, the teacher should use every means that will tend to *awaken thought.* Hence, there should be great variety in the examples, both as to their construction and phraseology, so as to prevent all mechanical ciphering according to fixed methods and rules.

The *Rules* and *Formulæ* given in this book are to be regarded as ***summaries*** to enable the pupil to retain processes previously analyzed and demonstrated. They need not be committed to memory, since the pupil will have acquired a sufficient knowledge of the principles involved to be able, at any time, to construct rules, if he has properly learned what precedes them.

In the higher department of arithmetic, the chief difficulty consists in giving the pupil a clear idea of the *nature* of the business transactions involved. The teacher should, therefore, strive by careful elucidation, to impart clear ideas of these transactions before requiring any arithmetical examples involving them to be performed. When the exact nature of the transaction is understood, the pupil's knowledge of abstract arithmetic will often be sufficient to enable him to solve the problem without any special rule.

The teacher should be careful not to advance too rapidly. The mind needs time to grasp and hold firmly every new case, and then additional time to bring its new acquisition into relation with those preceding it. Hence the need of frequent *reviews,* in order to give the pupil a comprehensive as well as an accurate and permanent knowledge of this subject.

The *Synopses for Review* interspersed through this work, are designed to afford assistance to the teacher in accomplishing this object. Each of these Synopses exhibits a brief, but definite, summary of all that is treated under the particular topic referred to, systematically and logically arranged, showing not only the different sub-topics, and their relations to each other and the general subject, but also the necessary preliminary definitions. Thus the teacher will be able *readily to ask an* exhaustive series of questions, without having *recourse to every* paragraph and page preceding.

Various useful exercises may be based upon these synopses. After the pupil has become familiar with their mode of construction, he may be required to write out, from memory, an *outline* synopsis of each section that he has studied, so as to show whether or not he has comprehended the relations of the various parts of the subject which he has passed over. Or the whole or a part of a Synopsis, embracing one or more topics, may be placed upon the blackboard, and the pupil required to give briefly but accurately the *sub-divisions*, *definitions*, *principles*, etc., involved in each. By this means, if further tested by questions, a thorough and well-classified knowledge of the whole subject will be permanently impressed upon his mind.

CONTENTS

PAGE
PERCENTAGE 1
  Profit and Loss 12
  Commission 20
  Synopsis 28
  Interest 29
  Problems in Interest 40
  Compound Interest 45
  Annual Interest 48
  Partial Payments 50
  Discount 54
  Bank Discount 57
  Savings Bank Accounts .. 62
  Synopsis 65
  Stocks 66
  Insurance 76
  Life Insurance 80
  Taxes 84
  Synopsis 88
  Exchange 89
  Foreign Exchange 93
  Arbitration of Exchange .. 98
  Custom-house Business ... 102
  Equation of Payments 105
  Averaging Accounts 110
  Synopsis 118
RATIO 119
PROPORTION 123
  Simple Proportion 126

PAGE
  Compound Proportion 131
  Partnership 137
ALLIGATION 143
  Synopsis 149
  Test Problems 150
INVOLUTION 155
EVOLUTION 161
  Cube Root 168
  Roots of Higher Degree .. 174
  Arithmetical Progression. 175
  Geometrical Progression .. 180
  Annuities 185
  Synopsis 189
MENSURATION 190
  Triangles 191
  Quadrilaterals 195
  Circles 197
  Similar Plane Figures 199
  Solids 203
  Prisms 203
  Pyramids and Cones 205
  Spheres 208
  Similar Solids 209
  Gauging 210
METRIC SYSTEM 212
  Synopsis 216
MEASURES AND TABLES 217
MIXED PROBLEMS 257

COMPLETE ARITHMETIC

SECOND PART.

PERCENTAGE

ORAL EXERCISES.

495. 1. What is $\frac{1}{100}$ of \$100? $\frac{5}{100}$? $\frac{25}{100}$? $\frac{75}{100}$?

2. What is $\frac{3}{100}$ of \$500? Of \$700? Of \$1000?

3. What is $\frac{5}{100}$ of \$600? $\frac{7}{100}$? $\frac{21}{100}$? $\frac{30}{100}$?

4. How many *hundredths* of \$100 are \$5? \$7? \$18?

5. How many hundredths of \$500 are \$25? \$35? \$50?

496. *Percentage* is a term applied to computations in which 100 is employed as a *fixed measure*, or standard.

497. *Per Cent.* is an abbreviation of the Latin phrase *per centum*, which signifies *by the hundred*.

Thus, 5 *per cent.* means 5 of every 100, or $\frac{5}{100}$, the 5 standing for the numerator, and the words "*per cent.*" for the denominator 100. Thus, 25 *per cent.* $= \frac{25}{100}$ or .25.

498. The ***Sign of Per Cent.*** is %. It is read *per cent.* *Thus 6% is* read 6 per cent.

510. **The base and rate being given to find the percentage.**

ORAL EXERCISES.

1. What is 10% of 140?

ANALYSIS.—10% is $\frac{10}{100}=\frac{1}{10}$, and $\frac{1}{10}$ of 140 is 14. Hence 10% of 140 is 14.

What is

2. 5% of $80?
3. 7% of 200 lb.?
4. 6% of 150 men?
5. 25% of 120 mi.?

How much is

6. 12½% of 72 gal.?
7. 40% of 60 sheep?
8. 8% of 50 bu.?
9. 50% of $240?

Find the amount

10. Of 100 A. + 27%.
11. Of $75 + 5%.
12. Of 32 doz. + 12½%.

Find the difference

13. Of 90 hhd. — 10%.
14. Of 63 Cd. — 33⅓%.
15. Of $200 — 2½%.

16. A farmer had 150 sheep, and sold 20% of them. How many had he left?

17. A mechanic who received $20 a week had his salary increased 8%. What were his daily wages then?

18. From a hhd. of molasses 33⅓% was drawn. How many gallons remained?

19. A grocer bought 150 dozen eggs, and found 16⅔% of them bad or broken. How many were salable?

20. A train of cars running 25 miles an hour increases its speed 12½%. How far does it then run in an hour?

511. PRINCIPLE.—*The percentage of any number is the same part of that number as the given rate is of* 100%.

WRITTEN EXERCISES.

512. 1. What is 17% of $4957 ?

OPERATION.

```
 $4957
   .17
$842.69
```

ANALYSIS.—Since 17% is .17, the required percentage is .17 of $4957, or $4957 × .17, which is $842.69.

What is

2. 35% of 695 lb. ?
3. 75% of $8428 ?
4. 12½% of £2105 ?

Find

5. 33⅓% of 8736 bu.
6. ½% of $35000.
7. 120% of $171.24.

RULE.—*Multiply the base by the rate. Or, take such a part of the base as the rate is of* 100%.

This rule may be briefly expressed by the following

FORMULA.—*Percentage* = *Base* × *Rate.*

What

8. Is 4½% of 312.8 rd.?
9. Is 105% of $5728?
10. Is $3140.75 + 1¼%?
11. Is 2⅔ mi. + 7½%?
12. Is 400 ft. − 3⅓%?

Find

13. 84% of 25⁴⁄₇ bu.
14. 25% of ⅞ of a ton.
15. ¾% of 16400 men.
16. ⅔% of ⅚ of a year.
17. ⅝% of ¹⁶⁄₂₅ of a hhd.

18. The bread made from a barrel of flour weighs 35% more than the flour. What is the weight of the bread?

19. A man having a yearly income of $4550 spends 20% of it the first year, 25% of it the second year, and 37½% of it the third year. How much does he save in 3 years?

20. A man receives a salary of $1600 a year. He pays 18% of it for board, 8⅓% for clothing, and 16% for incidentals. What are his yearly expenses, and what does he save?

21. A man owning $\frac{4}{5}$ of a cotton-mill, sold 35% of his share for $24640. What part of the whole mill did he still own, and what was its value?

22. Smith had $5420 in bank. He drew out 15% of it, then 20% of the remainder, and afterward deposited $12\frac{1}{2}$% of what he had drawn. How much had he then in bank?

513. The base and percentage being given to find the rate.

ORAL EXERCISES.

1. What per cent. of 25 is 3?

ANALYSIS.—Since 3 is $\frac{3}{25}$ of 25, it is $\frac{3}{25}$ of 100%, or 12%. Hence, 3 is 12% of 25.

What per cent.

2. Of 24 is 18?
3. Of $16 are $4?
4. Of 200 figs are 20 figs?
5. Of 40 lb. are 15 lb.?
6. Of $12\frac{1}{2}$ bu. are $2\frac{1}{2}$ bu.?
7. Of 2 A. are 80 sq. rd.?
8. Of 1 da. are 16 hr.?

What per cent.

9. Are $6\frac{1}{4}$ mi. of $12\frac{1}{2}$ mi.?
10. Are 18 qt. of 30 qt.?
11. Are $16\frac{2}{3}$ cents of $1?
12. Is $$\frac{1}{4}$ of $25?
13. Is $\frac{2}{3}$ of $\frac{8}{9}$?
14. Is $\frac{6}{7}$ of $2\frac{1}{3}$?
15. Is $\frac{3}{4}$ of $3\frac{3}{4}$?

16. $\frac{2}{3}$ of an acre is what per cent. of it?

17. $\frac{3}{5}$ of a cargo is what per cent. of it?

18. $2\frac{1}{4}$ times a number is what per cent. of it?

19. If $6 are paid for the use of $30 for a year, what is the rate per cent.?

20. If a milkman adds 1 pint of water to every gallon of milk he sells, what per cent. does he add?

514. PRINCIPLE.—*The rate is the number of hundredths which the percentage is of the base.*

WRITTEN EXERCISES.

515. 1. What per cent. of 72 is 48?

OPERATION.

$48 \div 72 = .66\frac{2}{3} = 66\frac{2}{3}\%$

Or, $\frac{48}{72} = \frac{2}{3}$; $100\% \times \frac{2}{3} = 66\frac{2}{3}\%$

ANALYSIS.—Since the percentage is the *product* of the base and rate, the rate is the quotient found by dividing the percentage by the base; and 48 divided by 72 is $\frac{48}{72} = \frac{2}{3} = .66\frac{2}{3}$; hence the rate is $66\frac{2}{3}\%$. Or,

Since 48, the percentage, is $\frac{2}{3}$ of the base, the rate is $\frac{2}{3}$ of 100%, or $66\frac{2}{3}\%$.

What per cent.
2. Of 300 is 75?
3. Of 66 is $16\frac{1}{2}$?
4. Of \$20 are \$21.60?

What per cent.
5. Of \$18 are 90 cents?
6. Of 560 lb. are 80 lb.?
7. Of 980 mi. are 49 mi.?

RULE.—*Divide the percentage by the base. Or, take such a part of* 100% *as the percentage is of the base.*

FORMULA.—*Rate = Percentage ÷ Base.*

What per cent.
8. Of \$480 are \$26.40?
9. Of 192 A. are 120 A.?
10. Of 15 mi. are 10.99 mi.?
11. Of 46 gal. are 5 gal. 3 qt.?
12. Of \$4 are 30 cents?
13. Of 6 bu. 1 pk. are 4 bu. 2 pk. 6 qt.?

What per cent.
14. Are 448 da. of 5600 da.?
15. Are 5 lb. 10 oz. of 15 lb. Avoir.?
16. Is 13.5 of 225?
17. Is $\frac{21}{35}$ of $\frac{56}{105}$?
18. Is $3\frac{5}{8}$ of $18\frac{1}{8}$?
19. Is $22\frac{4}{5}$ of 182.4?

20. A grocer sold from a hogshead containing 600 lb. of sugar, $\frac{1}{4}$ of it at one time, and $\frac{1}{3}$ of the remainder at another time. What per cent. of the whole remained?

21. A merchant owes \$15120, and his assets are \$9828. *What per cent. of* his debts can he pay?

516. The rate and percentage being given to find the base.

ORAL EXERCISES.

1. 18 is 3% of what number?

ANALYSIS.—Since 3%, or $\frac{3}{100}$, of a certain number is 18, $\frac{1}{100}$ is $\frac{1}{3}$ of 18, or 6, and $\frac{100}{100}$ is 600. Hence 18 is 3% of 600.

Of what number

2. Is 15 25%?

3. Is 24 75%?

4. Is 48 8%?

5. Is 1.2 6%?

Of what are

6. 30 lb. 20%? 25%?

7. \$84 12%? 21%?

8. 15 bu. 30%? 50%?

9. 16 doz. $12\frac{1}{2}$%? $8\frac{1}{3}$%?

10. $12\frac{1}{2}$% of 96 is $33\frac{1}{3}$% of what number?

517. PRINCIPLE.—*The base is as many times the percentage as* 100% *is times the rate.*

WRITTEN EXERCISES.

518. 1. 144 is 75% of what number?

OPERATION.

$$144 \div .75 = 192$$

$$\text{Or, } 100 \div 75 = \tfrac{100}{75} = \tfrac{4}{3}$$

$$144 \times \tfrac{4}{3} = 192$$

ANALYSIS.—Since the percentage is the *product* of the base by the rate, the base is equal to the percentage divided by the rate; and 144 ÷ .75 is 192. Or,

Since the rate is .75, the percentage is $\frac{75}{100}$, or $\frac{3}{4}$ of the base; hence the base is $\frac{4}{3}$ of the percentage, and $\frac{4}{3}$ of 144 is 192.

2. \$54 are 15% of what?

3. \$18.75 are $2\frac{1}{2}$% of what?

4. 4.56 A. are 5% of what?

5. 39.6 lb. are $7\frac{1}{2}$% of what?

RULE.—*Divide the percentage by the rate. Or, take as many times the percentage as* 100% *is times the rate.*

FORMULA.—*Base = Percentage ÷ Rate.*

Of what number
6. Is 828 120%?
7. Is 6119 105½%?
8. Is .43 71⅔%?
9. Is 31¼ 31¼%?

Of what
10. Are $281.25 37½%?
11. Are $4578 84%?
12. Are 37½ bu. 6¼%?
13. Are 1260 bbl. 12½%?

14. 25% of 800 bu. is 2½% of how many bushels?

15. A farmer sold 3150 bushels of grain and had 30% of his entire crop left. What was his entire crop?

16. A man drew 25% of his bank deposits, and expended 33⅓% of the money thus drawn in the purchase of a horse worth $250. How much money had he in bank?

17. If a man owning 45% of a steamboat sells 16⅔% of his share for $5860, what is the value of the whole boat?

18. If $295.12 are 13⅓% of A's money, and 4⅔% of A's money is 8% of B's, how much more money has A than B?

519. The amount, or the difference, and the rate being given to find the base.

ORAL EXERCISES.

1. What number increased by 25% of itself amounts to 60?

ANALYSIS.—Since 60 is the number increased by 25% of itself, it is $\frac{125}{100}$, or $\frac{5}{4}$ of the number; and if $\frac{5}{4}$ of the number is 60, the number itself is 4 times $\frac{1}{5}$ of 60, or 48.

2. What number increased by 8⅓% of itself is 130?

3. $70 are 40% more than what sum?

4. A man sold a saddle for $18, which was 12½% more than it cost him. What did it cost him?

5. A grocer sold flour for $8.40 a barrel, which was 16⅔% *more than he paid for it.* What did he pay for it?

6. What number diminished by 20% of itself is 40 ?

ANALYSIS.—Since 40 is the number diminished by 20% of itself, it is $\frac{80}{100}$, or $\frac{4}{5}$ of the number; and if $\frac{4}{5}$ of the number is 40, the number itself is 5 times $\frac{1}{4}$ of 40, or 50.

7. What number diminished by 5% of itself is 38 ?

8. What sum diminished by 50% of itself is $20.50 ?

9. 68 yd. are 15% less than what number ?

10. A tailor, after using 75% of a piece of cloth, had $9\frac{3}{4}$ yards left. How many yards in the whole piece ?

11. A sells tea at $.90 a pound, which is 10% less than he paid for it. What did he pay for it ?

WRITTEN EXERCISES.

520. 1. What sum increased by 37% of itself is $2050 ?

OPERATION.

$$1 + .37 = 1.37$$

$$\$2050 \div 1.37 = \$1500$$

Or,

$$\tfrac{100}{137} \text{ of } \$2050 = \overline{\$2050 \div 137} \times 100 = \$1500$$

ANALYSIS.—Since the number is increased 37%, or by .37 of itself, $2050 is 137%, or 1.37 the number. Hence $2050 divided by 1.37, is the base or required number. Or,

Since $2050, the amount, is $\frac{137}{100}$ of the base, 100 times $\frac{1}{137}$ of $2050, or $1500, is the base.

2. What number increased by 18% of itself equals 2950 ?

3. What sum increased by 15% of itself is $6900 ?

4. What number diminished by 12% of itself is 2640 ?

OPERATION.

$$1 - .12 = .88$$

$$2640 \div .88 = 3000$$

$$\text{Or, } \overline{2640 \div 22} \times 25 = 3000$$

ANALYSIS.—Since the number is diminished 12%, or by .12 of itself, 2640 is 88%, or .88 of the number. Hence 2640 divided by .88 is the base or required number. Or,

Since 2640, the difference, is $\frac{88}{100}$ or $\frac{22}{25}$ of the base, 25 times $\frac{1}{22}$ of *2640, or 3000,* is the base.

5. If the difference is \$1000 and the rate 20%, what is the base?

6. What sum diminished by 35% of itself equals \$4810?

RULE.—*Divide the amount by* 1 *plus the rate; or, divide the difference by* 1 *minus the rate.*

FORMULA.—$$Base = \begin{cases} Amount \div (1 + Rate). \\ Difference \div (1 - Rate). \end{cases}$$

What number increased

7. By 12% of itself is 3800?
8. By 10% is 39600?
9. By 15% is \$2616.25?
10. By 22% is 1098 bu.?

What number diminished

11. By 7½% of itself is 740?
12. By 4% is 312 acres?
13. By 8% is \$2281.60?
14. By 37½% is \$234.625?

15. A man sold 160 acres of land for \$4563.20, which was 8% less than it cost. What did it cost an acre?

16. A speculator bought 48 bales of cotton, and afterward sold the whole for \$2008.80, losing 7%. What was the cost of each bale?

17. A dealer bought a quantity of grain by measure and sold it by weight, thereby gaining 1½% in the number of bushels. He sold at 10% above the purchase price, and received \$4910.976 for the grain. Required the cost.

18. A merchant, after paying 60% of his debts, found that \$3500 would discharge the remainder. What was his whole indebtedness?

19. The net profits of a mill in two years were \$6970, and the profits the second year were 5% greater than the profits the first year. What were the profits each year?

20. A man sold two houses at \$2500 each; for one he received 20% more than its value and for the other 20% *less. Required his loss.*

APPLICATIONS OF PERCENTAGE.

521. The applications of percentage are those which are independent of *time*, as, Profit and Loss, Commission, Stocks, etc.; and those in which time is considered, as, Interest, Discount, Exchange, etc.

Since some one of the *four formulas* of percentage already considered will apply to any of these applications, the following will serve as a *general*

RULE.—*Note what elements of Percentage are given in the problem, and what element is required, and then apply the special rule or formula for the corresponding case.*

PROFIT AND LOSS.

522. ***Profit and Loss*** are terms used to express the gain or loss in business transactions.

523. Gains and losses are usually estimated at a *rate per cent.* on the cost, or the money or capital invested.

524. The operations involve the *same principles* as those of Percentage.

525. The corresponding terms are the following:

1. The ***Base*** is the *Cost*, or capital invested.
2. The ***Rate*** is the *per cent.* of profit or loss.
3. The ***Percentage*** is *profit* or *loss*.
4. The ***Amount*** is the cost *plus* the profit, or the *Selling Price.*
5. The ***Difference*** is the cost *minus* the loss, or the *Selling Price.*

ORAL EXERCISES.

526. 1. A horse that cost \$200 was sold at a gain of 12%. What was the gain, and the selling price?

ANALYSIS.—Since the gain was 12%, it was $\frac{12}{100}$ of \$200, which is \$24; and the selling price was \$200 + \$24 = \$224. Hence, etc. (**510.**)

2. A saddle that cost \$25 sold at a loss of 10%. What was the loss, and the selling price?

3. A tailor bought cloth at \$6 a yard, and wished to sell it at a gain of 25%. At what price must he sell it?

4. For how much must a grocer sell tea that cost \$.60 a pound, to gain 30%?

5. A merchant buys gloves at \$.75 a pair, and sells them at a profit of $33\frac{1}{3}\%$. For how much does he sell them?

6. Bought a carriage for \$160, and, after paying 10% for repairs, sold it at $12\frac{1}{2}\%$ profit. What was the gain, and the selling price?

7. If butter bought at 36 cents a pound is sold at a loss of $16\frac{2}{3}\%$, what is the selling price?

8. What must be the selling price of coffee that cost 25 cents a pound, in order to gain 20%?

9. At what price must an article that cost \$5 be sold, to gain 100%? 120%? 150%? 200%?

527. 1. A merchant bought cloth at \$5 a yard, and sold it at \$6 a yard. What was the gain per cent.?

ANALYSIS.—The whole gain is the difference between \$6 and \$5, which is \$1. Since \$5 gain \$1, or $\frac{1}{5}$ of itself, the gain per cent. is $\frac{1}{5}$ of 100% or 20%. Hence, etc. (**513.**)

2. What is gained per cent. by selling coal at \$7 a ton, that cost \$6 a ton?

3. Sold a piano for \$300, which was $\frac{3}{4}$ of what it cost. What was the loss per cent.?

4. Sold melons for $.75 that cost $.50. What was the gain per cent.?

5. What is gained per cent. by selling pine-apples at 30 cents each, that cost $15 a hundred?

6. Sold a sewing machine at a loss of $\frac{1}{8}$ of what it cost. What was the loss per cent.?

7. What % is gained on goods sold at *double* the cost?

8. What % is lost on goods sold at *one-half* the cost?

9. What per cent. profit does a grocer make who buys sugar at 10 cents and sells it at 12 cents?

10. What per cent. is gained on an article bought at $3 and sold at $5?

528. 1. A dealer sold flour at a profit of $2 a barrel, and gained 25%. What was the cost?

ANALYSIS.—Since the gain was 25% = $\frac{25}{100}$, or $\frac{1}{4}$, $2 is $\frac{1}{4}$ of the cost; $2 is $\frac{1}{4}$ of 4 times $2, or $8. Hence, etc. (**516.**)

2. Sold hats for $1 less than cost, and lost $16\frac{2}{3}$%. What did they cost?

3. A merchant sells silk at a profit of $$1\frac{1}{2}$ a yard, which is 40% gain. What did it cost, and what is the selling price?

4. If corn selling for 21 cents a bushel more than cost gives a profit of 30%, what did it cost?

5. Sold sheep at $$2\frac{1}{2}$ more than cost, which was a profit of 50%. What did they cost, and what is the selling price?

6. Shoes sold at $.50 above cost give a profit of $12\frac{1}{2}$%. What did they cost?

7. A farmer, by selling a cow for $12 less than she cost, lost $33\frac{1}{3}$%. What did she cost?

8. A grocer sells a certain kind of tea for 6 cents a *pound more than* cost and gains 5%. What did it cost?

529. 1. A watch was sold for $120, at a gain of 20%. What was the cost?

ANALYSIS.—Since the gain was 20%, or $\frac{1}{5}$, of the cost, $120, the selling price, is $\frac{6}{5}$ of the cost. $\frac{1}{6}$ of $120, or $20, is $\frac{1}{5}$ of the cost, and $\frac{5}{5}$, or the cost itself, is 5 times $20, or $100. Hence, etc. (**518.**)

2. Sold tea at $.90 a pound, and gained 25%. What did it cost?

3. A newsboy, by selling his papers at 4 cents each, gains 33⅓%. What do they cost him?

4. A man sold a horse and harness for $330, which was 10% more than they cost. What was their cost?

5. If 20% is lost by selling wheat at $1.60 a bushel, what would be gained if sold at 20% above cost?

6. John Rice lost 40% on a reaper, by selling it for $60. For what should he have sold it to gain 40%?

7. If, by selling books at $2 a volume, there is a gain of 25%, at what price must they be sold to lose 15%?

8. Two pictures were sold for $99 each; on one there was a gain of 10%, on the other a loss of 10%. Was there a gain or loss on the sale of both, and how much?

WRITTEN EXERCISES.

530. 1. A hogshead of sugar bought for $108.80 was sold at a profit of 12½%. What was the gain?

OPERATION.—$108.80 × .12½ = $13.16. (**512.**)

FORMULA.—*Profit* or *Loss* = *Cost* × *Rate* %.

Find the *Profit* or *Loss*,

2. On land that cost $1745, and was sold at a gain of 20%.

3. On goods that cost $3120, and were sold at 27% gain.

4. On a boat bought for $2545½, and sold at 25% loss.

5. On goods bought for $2560.75, and sold at 8% loss.

6. On 25 tons of iron rails bought at $58 a ton, and sold at an advance of 17½%.

7. A merchant pays $6840 for a stock of spring goods, and sells them at an advance of 26½% on the purchase price. After deducting $375 for expenses, what is his gain?

8. A miller bought 1000 bushels of wheat at $1.84 a bushel, and sold the flour at 16⅔% advance on the cost of the wheat. What was his profit?

9. Bought 128 tons of coal at $5.12½ a ton, and sold it at a gain of 22%. What was the entire profit?

10. A ship, loaded with 3840 bbl. of flour, being overtaken by a storm, found it necessary to throw 37½% of her cargo overboard. What was the loss at $7.65 a bbl.?

11. A man bought a pair of horses for $450, which was 25% less than their real value, and sold them for 25% more than their real value; what was his gain?

531. 1. Bought a house for $4380. For what must it be sold to gain 14½%?

OPERATION.—$\$4380 \times (1 + .14\tfrac{1}{2})$ or $1.145 = \$5015.10$. (**512.**)

2. At what price must pork, bought at $18.40 a barrel, be sold, to lose 15%?

OPERATION.—$\$18.40 \times (1 - .15)$, or $.85 = \$15.64$. (**512.**)

$$\text{FORMULA.—Selling Price} = \begin{cases} \text{Cost} \times (1 + \text{Rate \% of Gain}). \\ \text{Cost} \times (1 - \text{Rate \% of Loss}). \end{cases}$$

Find the *Selling Price*,

3. Of goods bought at $187.50, and sold at 11% gain.

4. Of beef bought at $12½ a barrel, and sold at 9½% loss.

5. Of cotton bought at $.14, and sold at a gain of 21⅖%.

6. Of cloth that cost $5¼ a yard, and was sold at a profit of 18½%?

7. At what price must goods that cost $3½ a yard be marked, to gain 25%? To lose 20%?

8. Sold a lot of damaged goods at a loss of 15%. What was the selling price of those that cost $.62½? $1.25?

9. Bought a hogshead of sugar containing 9 cwt. 56 lb. for $86.04, and paid $4.78 freight and cartage. At what price per pound must it be sold to gain 20%?

532. 1. Bought wool at $.48 a pound, and sold it at $.60 a pound. What per cent. was gained?

OPERATION.—$.60 − $.48 = $.12; and $.12 ÷ $.48 = .25 = 25%. (**515.**)

2. Sold for $10.02 an article that cost $12. What was the loss per cent.?

OPERATION.—$12 − $10.02 = $1.98; and $1.98 ÷ $12 = .16½ = 16½%.

FORMULA.—*Rate % = Profit* or *Loss ÷ Cost.*

Find the *rate per cent.* of profit or loss,

3. On sugar bought at 8 cents and sold at 9½ cents.

4. On tea bought at $1, and sold at $.87½.

5. On goods that cost $275, and were sold for $330.

6. On grain bought for $1.25 a bushel, and sold for $1.60 a bushel.

7. On a sewing-machine sold for $72.96, at a gain of $9.12.

8. On goods sold for $425.98, at a loss of $134.52.

9. Bought paper at $3 a ream, and sold it at 25 cents a quire. What was the gain per cent.?

10. A dealer bought 108 bbl. of apples at $4.62½, and sold them so as to gain $114.88½. What was his gain %?

11. If ½ of an acre of land is sold for ¾ the cost of an acre, what is the gain per cent.?

12. If $\frac{4}{5}$ of an acre of land is sold for the cost of $\frac{1}{2}$ of an acre, what is the loss per cent.?

13. If $\frac{5}{8}$ of a chest of tea is sold for what the whole chest cost, what is the gain per cent. on the part sold?

533. 1. A speculator sold grain at a profit of $33\frac{1}{3}\%$, by which he made 25 cents on a bushel. What did it cost?

OPERATION.—$\$.25 \div .33\frac{1}{3} = \$.75$. Or, $\$.25 \div \frac{1}{3} = \$.75$. (**518.**)

2. Lost \$45.75 on the sale of a horse, which was 20% of the cost. What was the cost?

OPERATION.—$\$45.75 \div .20 = \$228.75$. Or $\$45.75 \div \frac{1}{5} = \$228.75$.

FORMULA.—*Cost* = *Profit* or *Loss* ÷ *Rate* %.

Find the *Cost*,

3. Of goods sold at \$1500 profit, or a gain of 16%.

4. Of flour sold at a loss of \$.88, or 10%, on a barrel.

5. Of wheat sold at a loss of 6 cents, or 4%, on a bu.?

6. Of lumber sold at an advance of \$4.95 per M., or 35% gain.

7. If a grocer sells his stock at a profit of 15%, what amount must he sell to clear \$2500?

8. A and B engage in speculation. A gains \$2000, which is $12\frac{1}{2}\%$ of his capital, and B loses \$500, which is 5% of his capital. What sum did each invest?

534. 1. A furniture dealer sold two parlor sets for \$450 each; on one he made 15%, on the other he lost 15%. What did each cost him?

OPERATION.— $\begin{cases} \$450 \div (1 + .15) = \$391.30+, \text{ cost of one.} \\ \$450 \div (1 - .15) = \$529.41+, \text{ cost of the other.} \end{cases}$ (**520.**)

FORMULA.—*Cost* = *Selling Price* ÷ $\begin{cases} (1 + \textit{Rate}\ \%\ \text{of}\ \textit{gain.}) \\ (1 - \textit{Rate}\ \%\ \text{of}\ \textit{loss.}) \end{cases}$

Find the *Cost*,

2. Of coal sold at $6, being at a loss of 12½%.

3. Of grain sold at $.96 a bushel, at a gain of 28%.

4. Of silk sold for $5.40 a yard, at a profit of 10%.

5. Of hops sold at 16 cents a pound, at a loss of 20%.

6. Of fruit sold for $207.48, at a loss of 15%.

7. Having used a carriage 1 year, I sold it for $125, which was 25% below cost. What should I have received had I sold it for 10% above cost?

8. B sold a span of horses to C and gained 12½%; C sold them to D for $550, and lost 16⅔%. What did the horses cost B?

9. If a piece of property increases in value each year at the rate of 25% on the value of the previous year, for 4 years, and then is worth $16000, what did it cost?

535. 1. Bought cloth at $3.60 a yard. At what price must it be marked that 12½% may be abated from the asking price, and still a profit made of 16⅔%?

OPERATION.— $\begin{cases} \textit{Selling Price} = \$3.60 \times (1 + .16\frac{2}{3}) = \$4.20. \\ \textit{Marking Price} = \$4.20 \div (1 - .12\frac{1}{2}) = \$4.80. \end{cases}$ (**519.**)

2. At what price must shovels that cost $1.12 each be marked in order to abate 5%, and yet make 25% profit?

3. How must a watch be marked, that cost $120, so that 4% may be deducted and a profit of 20% be made?

4. A merchant, on opening a case of goods that cost $.80 a yard, finds them slightly damaged. How must he mark them, to fall 25% in his asking price, and sell at cost?

5. Bought land at $60 an acre; how much must I ask an acre, that I may abate 25% from my asking price, and still make 20% on the purchase price?

COMMISSION.

536. An ***Agent*** or ***Commission Merchant*** is a person who buys or sells merchandise, or transacts other business for another, called the *Principal.*

537. ***Commission*** is the fee, or compensation, allowed an agent or commission merchant for transacting business, and is usually computed at a certain rate per cent. of the money involved in the transaction.

538. A ***Consignment*** is a quantity of goods sent to a commission merchant to be sold.

539. The ***Consignor*** is the person who sends the goods for sale. A consignor is sometimes called a *Shipper*.

540. The ***Consignee*** is the person to whom the goods are sent. He is sometimes called a *Correspondent.*

541. The ***Net Proceeds*** of a sale or other transaction is the sum of money that remains after all expenses of commission, etc., are paid.

542. A ***Guaranty*** is security given by a commission merchant to his principal for the payment of goods sold by him on credit.

543. An ***Account Sales*** is a written statement made by a commission merchant to his principal, containing an account of goods sold, their price, the expenses, and the net proceeds.

544. A ***Broker*** is a person who buys or sell stocks, bills of exchange, real estate, etc., for a commission, which is called *Brokerage.*

545. The *principles* and *operations* of Percentage involved in Commission and Brokerage are the same as those already treated.

546. The following are the corresponding terms:

1. The ***Base*** is the amount of sales, money invested, or collected.

2. The ***Rate*** is the per cent. allowed for services.

3. The ***Percentage*** is the Commission or Brokerage.

4. The ***Amount*** or ***Difference*** is the amount of sales, *plus* or *minus* the commission.

WRITTEN EXERCISES.

547. Find the *Commission* or *Brokerage*,

1. On a sale of flour for $2575, at 2½%.

OPERATION.—$2575 × .025 = $64.37½. (**512.**)

FORMULA.—*Amount of Sales* × *Rate* % = *Commission.*

2. On the purchase of a farm for $13750, at 2¾%.

3. On the sale of a mill for $9384, at ⅞%.

4. On the sale of 21680 lb. of wool, at 1¾ cents a pound.

5. On the sale of 250 bales of cotton, averaging 520 lb., at 14⅝ cents a pound; commission 1½%.

6. On the sale of 175 shares of stock, at $92¼ a share; brokerage, ⅛%.

7. On the sale at auction of a house and the furniture for $9346.80, at 6¼%.

8. A commission merchant sells 225 bbl. of potatoes at $3.25 per bbl., and 316 bbl. of apples at $4½ per bbl. What is his commission at 4½%?

548. Find the *rate* of commission or brokerage,

1. When $89 commission is paid for selling goods for $3560.

OPERATION.—89 ÷ 3560 = .02½ = 2½%. (**515.**)

FORMULA.—*Commission ÷ Amount of Sales = Rate %.*

2. When $165 com. is paid for selling goods for $4950.

3. When $63 is paid for collecting a debt of $1260.

4. When $117.75 is paid for selling a house for $7850.

5. When $235.40 is paid for buying 26750 lb. of wool at 32 cents a pound.

6. When $125 is paid for the guaranty and sale of goods for $2500.

7. Paid my N. O. agent $74.25 for buying 26400 lb. of rice, at 4½ ct. a lb. What was the rate of his commission?

549. Find the *Amount of Sales,*

1. When a commission of $147 is charged at 3½%.

OPERATION.—$147 ÷ .035 = $4200. (**517.**)

FORMULA.—*Commission ÷ Rate % = Amount of Sales.*

2. When $92.80 commission is paid at 3⅓%.

3. When $210 commission is charged at 6%.

4. When $24 brokerage is paid at ¼%.

5. When $135 commission is charged at 1½%.

6. Paid an attorney $72.03 for collecting a note, which was a commission of 7½%. What was the face of the note?

550. Find the *Amount of Sales,*

1. When the net proceeds are $4875, commission 2½%.

OPERATION.—$4875 ÷ .975 = $5000. (**519.**)

FORMULA.—*Net proceeds ÷ (1 — Rate %) = Amt. of Sales.*

2. When the net proceeds are $3281.25, commission 12½%.

3. When the net proceeds are $560, and the com. 4%.

4. After deducting 6¼% commission and $132 for storage, my correspondent sends me $23654.25 as the net proceeds of a consignment of pork and flour. What was the gross amount of the sale?

551. Find the *amount to be invested*,

1. If $9500 is remitted to a correspondent to be invested in woolen goods, after deducting 5% commission.

OPERATION.—$9500 ÷ 1.05 = $9047.62. (**519.**)

FORMULA.—*Amount Remitted* ÷ (1 + *Rate %*) = *Sum Invested.*

2. If $4908 be remitted, deducting 4½% commission.

3. If $3246.20 be remitted, deducting 2% commission.

4. If $1511.25 be remitted, deducting ¾% commission.

5. If $10701.24 be remitted, deducting ½% brokerage.

6. A dealer sends his agent in Havana $6720.80, with which to purchase oranges and other fruits, after deducting his commission of 5%. What sum did the agent invest, and what was the amount of his commission?

7. What amount of sugar can be bought at 8 cents a pound, for $2523.40, after deducting a commission of 1¾%.

8. Remitted to a stockbroker $10650, to be invested in stocks, after deducting ¼% brokerage. What amount of stock did he purchase?

9. A broker received $45337.50 to invest in bond and mortgage, after deducting a commission of 2½%. What amount did he invest, and what was his commission?

10. Sent $250.92 to my agent in Boston, to be invested in prints at 15 cents a yard, after taking out his commission of 2%. How many yards ought I to receive?

REVIEW.

ORAL EXERCISES.

552. 1. If stoves bought at $36 each are sold at a profit of 8⅓%, what is the gain?

2. What will be the expense of collecting a tax of $1000, allowing 5%?

3. What will a broker receive for selling $600 worth of stock, at ⅜% brokerage?

4. A man having $250 spent $80. What per cent. of his money had he left?

5. If a man sells a building lot that cost $300, at an advance of 166⅔%, what is his gain?

6. ⅗ of 30% is what per cent. of 72%? Of 144%? Of 180%? 240%?

7. Bought a horse for 20% less than $200, and sold him for 10% more than $200. What per cent. was gained?

8. How many bushels of wheat at $2 a bushel can an agent buy for $2040, and retain 2% on what he expends as his commission?

9. If by selling land at $150 an acre I lose 25%, how must I sell it to gain 40%?

10. A boy bought bananas for $3 a hundred, and sold them for 5 cents each. What per cent. did he gain?

11. Bought cannel coal at $19 a ton, which was 5% less than the market price. What was the market price?

12. Paid an agent $150, or a commission of 1½%, for selling my house. For what sum was the house sold?

13. If an article is sold so as to gain ⅖ as much as it cost, what per cent. is gained?

14. A merchant tailor sold some linen coats at $1.80 each, which was 33⅓% below the marked price. What was the marked price?

15. A grocer bought 40 gal. of maple syrup at the rate of 4 gal. for $6, and sold it at the rate of 5 gal. for $8. What was his whole gain, and his gain per cent.?

16. How much wheat must a farmer take to mill that he may bring away the flour of 4½ bushels, after the miller takes his toll of 10%?

WRITTEN EXERCISES.

553. 1. After taking out 15% of the grain in a bin, there remained 40 bu. 3⅕ pk. How many bushels were there at first?

2. The net profits of a farm in 2 years were $3485, and the profits the second year were 5% greater than the profits the first year. What were the profits each year?

3. A has 32% more money than B; what per cent. less is B's money than A's?

4. Bought 450 bushels of wheat at $1.25 per bushel, and sold it at $1.40 per bushel. What was the whole gain, and the gain per cent.?

5. A man drew out of the bank ⅗ of his money, and expended 30% of 50% of this for 728 bu. of wheat, at $1.12½ a bushel. What sum had he left in bank?

6. Sold goods to the amount of $47649, at a profit of 16⅔%. Required the cost and the total gain.

7. A broker received $37.50 for selling some uncurrent money, charging ⅛% brokerage. How much did he sell?

8. If ⅘ of a farm is sold for what ⅝ of it cost, what is the gain per cent.?

9. An architect charged ¼% for plans and specifications, and 1⅝% for superintending a building that cost $25000. What was the amount of his fee?

10. If a stationer marks his goods 50% above cost, and then abates 50%, what per cent. does he make or lose?

11. Sold a farm for $14700, and lost 12%. What per cent. should I have gained by selling it for $21000?

12. If an article bought at 20% below the asking price is sold at 16% below that price, what is the rate of gain?

13. A commission merchant sold a consignment of goods for $5250, and charged 3¼% commission, and 2½% for a guaranty. Find the net proceeds.

14. Smith & Jones bought a stock of groceries for $13680. They sold ¼ of the entire stock at 15% profit, ⅓ at 18¾%, ⅙ at 20%, and the remainder at 33⅓% profit. What was the whole gain, and the average gain per cent.?

15. Give the marking prices at 25% advance, of the following bill of goods, and the amount when sold at an abatement of 10% from those prices:

1 Case of Prints,	450 yd.,	@ $.12
3 Pieces Cassimeres,	65 "	@ 3.25
1 Bale Ticking,	244 "	@ .20
25 Dress Shawls,		@ 7.36
1 Gr. gross Clark's Thread,	144 doz.,	@ .70
50 Gross Buttons,		@ 1.00

16. How much would the above bill of goods amount to if sold at 5½% below a marking price of 15% above cost?

17. What would be the net proceeds of a sale of 18 cwt. 75 lb. of sugar, at $9⅖ per cwt., allowing 2½% commission, *and $16¾ for* other charges?

18. A broker receives $7125 to invest in cotton, at 11¼ cents a pound. If his commission is 2½%, how many pounds of cotton can he buy?

19. If the sale of potatoes at $.75 a barrel above cost gives a profit of 18¾%, how much must be added to this price to realize a profit of 31¼%?

20. An agent in Chicago purchases 1000 bbl. of flour at $6.80, and pays 5 cents a barrel storage for 30 days; also, 3000 bu. of wheat at $1.20. He charges a commission of 1½% on the flour, and 1 cent a bushel on the wheat. What sum of money will balance the account, and what is the amount of his commission?

21. An agent in Boston received 28000 lb. of Texas cotton, which he sold at $.12½ a pound. He paid $45.86 freight and cartage, and after retaining his commission, he remits his principal $3252.89 as the net proceeds of the sale. What was the rate of his commission?

22. The following bill of goods was sold at auction:

1½ bbl. A Sugar,	312 lb.,	@ $.12½	that	cost	$.11¼
½ " Pulv. "	96 "	@ .14⅛	"	"	.14
1 Chest Y. H. Tea,	84 "	@ 1.10	"	"	1.12½
1 Box Soap,	60 "	@ .13	"	"	.10¾
1½ Sacks Java Coffee,	110 "	@ .22½	"	"	.24¼
184 lb. Codfish,		@ .07½	"	"	.08⅜

Allowing a commission of 4½% for selling, find the entire profit or loss, and the gain or loss per cent. on the whole.

23. A merchant in New York imported 2400 yd. of English cloth, for which he paid in London 10s. sterling a yard, and the total expenses were $255. He sold the cloth for $3.81 a yard, U. S. money. What was his whole gain, and his gain per cent.?

554. SYNOPSIS FOR REVIEW.

PERCENTAGE.

- 1. DEFINITIONS.
  - 1. Percentage. 2. Per Cent. 3. Sign of Per Cent. 4. Rate, or Rate %. 5. Base. 6. Percentage. 7. Amount. 8. Difference.
- 2. ELEMENTS.
  - 1. How many considered.
  - 2. How many must be given.
- 3. **510.** 1. Principle. 2. Rule. 3. Formula.
- 4. **513.** 1. Principle. 2. Rule. 3. Formula.
- 5. **516.** 1. Principle. 2. Rule. 3. Formula.
- 6. **519.** 1. Principle. 2. Rule. 3. Formula.
- 7. APPLICATIONS OF PERCENTAGE.
  - 1. Diff't kinds.
    - 1. *Without Time.*
    - 2. *With Time.*
  - 2. General Rule.
- 8. PROFIT AND LOSS.
  - 1. Definition.
  - 2. To estimate gains and losses.
  - 3. Corresponding terms.
    - 1. *Base.*
    - 2. *Rate.*
    - 3. *Percentage.*
    - 4. *Am't and Diff.*
- 9. COMMISSION.
  - 1. Definitions.
    - 1. *Agent, or Commission Merchant.*
    - 2. *Commission.*
    - 3. *Consignment.*
    - 4. *Consignor.*
    - 5. *Consignee.*
    - 6. *Net Proceeds.*
    - 7. *Guaranty.*
    - 8. *Account Sales.*
    - 9. *Broker.*
  - 2. Prin. and Operations Involved.
  - 3. Corresponding terms.
    - 1. *Base.*
    - 2. *Rate.*
    - 3. *Percentage.*
    - 4. *Am't and Diff.*

INTEREST

ORAL EXERCISES.

555. 1. When 5% is charged for the use of money, how many dollars should be paid for the use of $100? For the use of $200? Of $500? Of $50?

2. At 7% a year, what should be paid for the use of $100 for 2 years? Of $200 for 3 years?

3. If $500 is loaned for 3 years, what should be paid for its use, at 5% a year? At 6% a year?

4. If I borrow $250, and agree to pay 4% a year for its use, how much will be due the lender in 5 years?

5. If $7 is paid for the use of $100 for 1 year, what is the per cent.?

6. If $50 is paid for the use of $100 for 5 years, what is the per cent.?

7. If $14 is paid for the use of $200 for 1 year, what is the per cent.?

8. At 6%, what decimal part of the money borrowed is equal to the money paid for its use? At 7%? 8%? 9%?

DEFINITIONS.

556. ***Interest*** is a sum paid for the use of money.

557. The ***Principal*** is the sum for the use of which interest is paid.

558. The ***Rate of Interest*** is the per cent., or number of hundredths, of the principal, paid for its use for *one year.*

559. The ***Amount*** is the sum of the principal and the interest.

560. ***Legal Interest*** is the interest according to the rate per cent. *fixed by law.*

561. ***Usury*** is a higher rate of interest than is allowed by law.

562. The *legal* rates of interest in the different States are as follows:

Name of State.	Rate.		Name of State.	Rate.	
Alabama	8%	......	Minnesota	7%	12%
Arkansas*	6%	Any.	Mississippi	6%	10%
Arizona	10%	Any.	Missouri	6%	10%
California*	10%	Any.	Montana	10%	......
Canada and Ireland	6%	......	New Hampshire.	6%	......
Connecticut	7%	......	New Jersey	7%	.....
Colorado*	10%	Any.	New York	7%	......
Dakota.	7%	Any.	North Carolina.	6%	8%
Delaware.	6%	......	Nebraska.	10%	15%
Dist. Columbia.	6%	10%	Nevada*.	10%	Any.
England and France	5%	......	Ohio	6%	8%
Florida*.	8%	Any.	Oregon	10%	12%
Georgia	7%	10%	Pennsylvania	6%	7%
Idaho.	10%	......	Rhode Island*.	6%	Any.
Illinois	6%	10%	South Carolina*.	7%	Any.
Indiana	6%	10%	Tennessee	6%	10%
Iowa	6%	10%	Texas.	8%	12%
Kansas	7%	2%	Utah*.	10%	Any.
Kentucky	6%	10%	Vermont	6%	......
Louisiana.	5%	8%	Virginia.	6%	12%
Maine*.	6%	Any.	West Virginia.	6%	......
Maryland	6%	......	Washington T.*	10%	Any.
Massachusetts*.	6%	Any.	Wisconsin.	7%	10%
Michigan.	7%	10%	Wyoming	12%	......

1. When the rate per cent. is not specified in accounts, notes, mortgages, contracts, etc., the *legal* rate is always understood.

2. Where *two* rates are specified, any rate above the lower, and not exceeding the higher, is allowed, *if stipulated in writing.*

3. In the States marked thus (*) the rate per cent. is unlimited if agreed upon by the parties in writing.

563. In the operations of interest there are *five* parts, or elements, namely:

The *Principal;* the *Rate per Cent. per Annum* (for one year); the *Interest;* the *Time* for which the principal is lent; and the *Amount*, or sum of the Prin. and Int.

564. These terms correspond respectively to *Base*, *Rate*, *Percentage*, and *Amount* in Percentage, excluding *Time*, which is an additional element in Interest.

ORAL EXERCISES.

565. 1. At 3% for 1 yr., what decimal part of the principal equals the interest? At 5%? At 8%? At 12½%?

2. What is the interest of $20 for 1 year at 5%?

ANALYSIS.—Since the interest of any sum at 5% for 1 yr. is .05 of the principal, the interest of $20 for 1 yr. at 5% is .05 of $20, or $1.

3. What is the interest of $50 for 1 yr. at 5%? 6%? 7%?

4. What is the interest of $80 for 1 yr. at 7%? 8%? 10%?

5. At 7% for 5 yr., what decimal part of the principal equals the interest?

ANALYSIS.—Since the interest at 7% for 1 yr. is .07 of the principal, the interest for 5 yr. is 5 times .07, or .35 of the principal. Or, it is 5 times the interest for 1 year.

6. At 6% for 3 yr., what decimal or fractional part of the principal equals the interest? At 7% for 6 yr.? At 5% for 5 yr.? At 6¼% for 2 yr.? At 10% for 4 yr.?

7. Find the interest of $30 for 3 yr. at 5%.

ANALYSIS.—Since the interest of any sum at 5% for 1 yr. is .05 of the principal, for 3 yr. it is .15, and .15 of $30 is $4.50. Or, the interest for 1 yr. is .05 of $30, or $1.50, and for 3 yr. it is 3 times as much, or $4.50.

8. Find the int. at 6% of $20 for 2 yr. Of $40 for 3 yr.

9. Find the int. at 8% of $5 for 5 yr. Of $10 for 10 yr.

10. At 8% for 2 yr. 6 mo., what decimal part of the principal equals the interest?

ANALYSIS.—Since the interest of any sum for 1 yr. at 8% is .08 of the principal, the interest on the same for 2 yr. 6 mo. is 2½ times .08, or .20 of the principal. Or, it is 2½ times the interest for 1 year.

11. At 6% for 3 yr. 3 mo., what decimal part of the principal equals the interest? At 9% for 3 yr. 3 mo.?

12. Find the int. of $9 for 2 yr. 4 mo. at 7%. At 8%.

13. What is the int. of $1000 for 2 yr. 3 mo. at 10%? For 4 yr. 6 mo.? For 5 yr. 3 mo.? For 8 mo.?

566. PRINCIPLE.—*The interest is the product of three factors; namely, the principal, rate per annum, and time (expressed in years or parts of a year).*

WRITTEN EXERCISES.

567. To find the interest or amount of any sum, at any rate per cent., for years and months.

1. Find the amount of $97.50, at 7%, for 2 yr. 6 mo.

OPERATION.

$97.50	
.07	
$6.8250	Int. for 1 yr.
2½	
17.0625	Int. for 2 yr. 6 mo.
97.50	Principal.
$114.5625	Amount.

ANALYSIS.—Since the interest of any sum at 7% for 1 yr. is .07 of the principal, the interest of $97.50 at 7% for 1 yr. is .07 of $97.50, or $6.825; and the interest for 2 yr. 6 mo. is 2½ times the interest for 1 yr., or $17.06¼, and $17.06¼ + $97.50 = $114.56¼, the *Amount.*

Find the interest and the amount,

2. Of $450 for 3 yr. 9 mo. at 6%. For 8 mo. at 7%.

3. Of $247 for 5 yr. 3 mo. at 5½%. For 10 mo. at 8%.

4. Of $500 for 4 yr. 2 mo. at 10%. For 11mo. at 5%.

RULE.—I. *Multiply the principal by the rate, and the product is the interest for* 1 *year.*

II. *Multiply the interest for* 1 *year by the time in years, and the fraction of a year; the product is the required interest.*

III. *Add the principal to the interest for the amount.*

FORMULA.—*Interest* = *Principal* × *Rate* × *Time.*

Find the interest,

5. Of $36.40 for 1 yr. 7 mo. at 6%. At 7%. At 7½%.
6. Of $750.50 for 3 yr. 1 mo. at 5%. At 8%. At 9%.
7. Of $1346.84 for 2 yr. 4 mo. at 6¼%. At 7½%.
8. Of $138.75 for 4 yr. 3 mo. at 10%. At 12½%.
9. Find the amount of $640 for 5 yr. 6 mo. at 7%.
10. Find the amount of $56.64 at 8% for 3 yr. 3 mo.
11. Made a loan of $1040 for 1 yr. 9 mo. at 7½%. How much is due at the end of the time?
12. If a note for $375, on interest at 8%, dated June 10, 1874, be paid Sept. 10, 1876, what amount will be due?

568. To find the interest on any sum of money, for any time, at any rate per cent.

OBVIOUS RELATIONS BETWEEN TIME AND INTEREST.

I. The interest on any sum for 1 year at 1% is .01 of the principal.

It is therefore equal to the principal with the decimal point removed *two places* to the left.

II. The interest for 1 mo. is $\frac{1}{12}$ of the interest for 1 yr.

III. The interest for 3 days is $\frac{3}{30}$, or $\frac{1}{10}$, of the interest for 1 month; hence any number of days may readily be reduced to *tenths* of a month by dividing by 3.

IV. The interest on any sum for 1 month, multiplied by the number of *months* and *tenths* of a month in the given time, and the product by the number expressing the *rate*, will be the required interest.

569. 1. Find the int. of $361.20 for 1 yr. 3 mo. 24 da. at 7%.

OPERATION.

$3.612	(.01 of the Prin.) Int. for 1 yr. at 1% (**568**, I).
.301	Int. for 1 mo. at 1% (**568**, II).
15.8	Number of months and tenths (**568**, III).
$4.7558	Int. for 1 yr. 3 mo. 24 da. at 1%.
7	
$33.2906	Int. for 1 yr. 3 mo. 24 da. at 7% (**568**, IV).

What is the interest,

2. Of $137.25 for 1 yr. 6 mo. 10 da. at 6%? At 4%?
3. Of $510.50 for 3 yr. 7 mo. 15 da. at 5%? At 8%?
4. Of $1297.60 for 2 yr. 11 mo. 18 da. at 7%? At 7½%?

RULE.—I. To find the interest for 1 yr. at 1%.

Remove the decimal point in the given principal two places to the left.

II. To find the interest for 1 mo. at 1%.

Divide the interest for 1 *year by* 12.

III. To find the interest for *any time* at 1%.

Multiply the interest for 1 *month by the number of months and tenths of a month in the given time.*

IV. To find the interest at any rate %.

Multiply the interest at 1% *for the given time by the number expressing the given rate.*

5. Find the int. of $781.90 for 1 yr. 1 mo. 12 da. at 7%.
6. Find the int. of $3000 for 11 mo. 21 da. at 10%.

7. What is the amt. of \$1049 for 2 yr. 3 mo. 9 da. at $6\frac{1}{2}\%$?

8. What is the amt. of \$216.75 for 3 yr. 5 mo. 11 da. at 8%?

9. Required the int. of \$250 from Jan. 1, 1873, to May 10, 1875, at 7%?

10. Required the amount of \$408.60 from Aug. 20 to Dec. 18, 1876, at 10%?

11. What is the interest on a note for \$515.62, dated March 1, 1873, and payable July 16, 1875, at 7%?

12. A man sold his house and lot for \$12500; the terms were, \$4000 in cash on delivery, \$3500 in 9 mo., \$2600 in 1 yr. 6 mo., and the balance in 2 yr. 4 mo., with 6% interest. What was the whole amount paid?

570. SIX PER CENT METHOD.

At 6% per annum, the interest of \$1

For	12 mo.		is 6 cents,	or .06	of the	principal.
"	2 "	or $\frac{1}{6}$ of 12 mo.,	" 1 cent,	" .01	"	"
"	1 "	" $\frac{1}{12}$ " 12 "	" $\frac{1}{2}$ "	" .005	"	"
"	6 da.	" $\frac{1}{5}$ " 1 "	" $\frac{1}{10}$ "	" .001	"	"
"	1 "	" $\frac{1}{6}$ " 6 da.	"	.000$\frac{1}{6}$	"	"

571. PRINCIPLES.—1. *The interest of any sum at 6% is* ONE-HALF *as many hundredths of the principal as there are* MONTHS *in the given time.*

2. *The interest of any sum at 6% is* ONE-SIXTH *as many thousandths of the principal as there are* DAYS *in the given time.*

Thus, the interest on any sum at 6% for 1 yr. 3 mo., or 15 mo., is $\frac{1}{2}$ of .15, or .075, of the principal; and for 18 da. it is $\frac{1}{6}$ of .018, or .003, of the principal. Hence, for 1 yr. 3 mo. 18 da., it is .075 + .003 = .078 of the principal.

It is evident that an odd month is $\frac{1}{2}$ of .01, or .005; and that any number of days less than 6 is such a fractional part of .001 as the days are of 6 days.

ORAL EXERCISES.

572. What is the interest,

1. Of $1 at 6% for 1 year? 2 years? 3 years? 5 years? 8 years? 12 years?

2. Of $1 at 6% for 1 month? 2 mo.? 3 mo.? 4 mo.? 5 mo.? 7 mo.? 9 mo.? 10 mo.? 15 mo.? 18 mo.?

At 6%, what is the interest,

3. Of $1 for 1 yr. 4 mo.? 1 yr. 7 mo.? 2 yr. 2 mo.?

4. Of $1 for 1 day? 6 da.? 12 da.? 19 da.? 24 da.? 33 da.? 36 da.? 45 da.? 63 da.?

5. Of $1 for 1 mo. 12 da.? For 3 mo. 15 da.? For 6 mo. 25 da.? For 7 mo. 11 da.? For 11 mo. 18 da.?

Find the interest,

6. Of $1, at 6%, for 1 yr. 3 mo. 6 da. For 1 yr. 9 mo. 18 da. For 1 yr. 5 mo. 19 da.

7. Of $1 at 6% for 2 yr. 1 mo. 9 da. For 3 yr. 24 da.

8. Of $1 at 6% for 5 yr. 5 mo. 5 da. For 4 yr. 7 mo. 10 da.

At 6%, find the interest,

9. Of $1 for 2 yr. 6 mo. Of $2. Of $3. Of $5.

10. Of $1 for 4 yr. 2 mo. Of $10. Of $20. Of $30.

11. Of $5 for 1 yr. 4 mo. For 2 yr. For 2 yr. 8 mo.

12. Of $1 for 33 da. For 63 da. For 93 da. For 123 da.

13. Of $6 for 33 da. Of $4 for 63 da. Of $2 for 93 da.

14. If the interest of a certain principal at 6% is $18, what would the interest be at 5%? 7%? 8%? 9%?.

5% is ⅙ less than 6%; 7% is ⅙ more than 6%; 8% is ⅓ more, etc.

15. If the interest of a certain principal is $16, what would the int. be at 3%? 4½%? 5%? 7½%? 8%? 12%?

16. If the interest of a certain principal is $30, what would the int. be at 2%? 4%? 7%? 8%? 10%? 14%?

WRITTEN EXERCISES.

573. 1. What is the int. of $427.20 at 6% for 2 yr. 5 mo. 27 da.

OPERATION.

2 yr. 5 mo. = 29 mo. $427.20
½ of .29 = .145 .149½
⅙ of .027 = .004½ $63.8664
Int. = .149½ of the Prin.

ANALYSIS.—Since the interest of $1 for 2 yr. 5 mo. 27 da. is $.149½, or of any sum is .149½ of the principal (**571**), $427.20 × .149½ = $63.866 + is the required interest.

Find the interest at 6% of

2. $597.25 for 7 mo. 18 da.
3. $418.75 for 1 mo. 25 da.
4. $309.18 for 2 yr. 24 da.
5. $1298 for 3 yr. 1 mo. 13 da.
6. $2000 for 2 yr. 7 mo. 24 da.
7. $4010 for 1 yr. 1 mo. 13 da.

RULE.—*Multiply the given principal by the decimal expressing the interest of* $1; *or by the decimal expressing one-half as many hundredths as there are months, and one-sixth as many thousandths as there are days, in the given time, and the product will be the required interest.*

To find the interest at any other per cent. by this method, increase or diminish the interest at 6% by such part of itself as the given rate is greater or less than 6%.

574. To compute ***Accurate Interest,*** that is, reckoning 365 da. to the year, use the following

RULE.—*Find the interest for years and aliquot parts of a year by the common method, and for days take such part of* 1 *year's interest as the number of days is of* 365. Or,

When the time is in days and less than 1 *year, find the interest by the common method and then subtract* $\frac{1}{73}$ *part of itself for the common year, or* $\frac{1}{61}$, *if it be a leap year.*

1. Find the accurate interest of \$1560 for 45 da. at 7%.

The *exact int.* of \$1560 for 45 da. at 7% $= \frac{\$109.20 \times 45}{365} = 13.46+$.

Or, it is $\$13.65 - \frac{\$13.65 \times 1}{73} = \$13.46 +$.

2. Find the exact int. of \$1600 for 1 yr. 3 mo. at 6%.

3. What is the difference between the *exact* interest of \$648.40 at 8% for 1 yr. 3 mo. 20 da. and the interest reckoned by the 6% method.

4. Find the exact interest of \$875.60 at 7% for 63 da.?

5. Required the exact interest on three U. S. Bonds of \$1000 each, at 6%, from May 1 to Oct. 15.

6. What is the exact interest on a \$500 U. S. Bond, at 5%, from Nov. 1 to April 10 following?

575. Find the interest, by any of the ordinary methods,

1. Of \$721.56 for 1 yr. 4 mo. 10 da. at 6%.
2. Of \$54.75 for 3 yr. 24 da. at 5%.
3. Of \$1000 for 11 mo. 18 da. at 7%.
4. Of \$3046 for 7 mo. 26 da. at 8%.
5. Of \$1821.50 from April 1 to Nov. 12 at 6%.
6. Of \$700 from Jan. 15 to Aug. 1 at 10%.
7. Of \$316.84 from Oct. 20 to March 10 at 7%.

What is the amount

8. Of \$3146 for 2 yr. 3 mo. 10 da. at 7%?
9. Of \$96.85 for 3 yr. 1 mo. 27 da. at 6%?
10. Of \$1008.80 for 10 mo. 16 da. at 6½%?
11. Of \$2000 for 15 da. at 12½%?
12. Of \$137.60 for 127 da. at 10%?
13. If \$1671.64 be placed at interest June 1, 1874, what amount will be due April 1, 1876, at 7%?

14. How much is the interest on a note for $600, dated Feb. 1, 1872, and payable Sept. 25, 1875, at 6%?

15. If a man borrow $9700 in New York, and loan it in Colorado, what will it gain at legal int. in a year?

16. Required the interest of $127.36 from Dec. 12, 1873, to July 3, 1875, at 4½%.

17. A note of $250, dated June 5, 1874, was paid Feb. 14, 1875, with interest at 8%. What was the amount?

18. A note for $710.50, with interest after 3 mo., at 7%, was given Jan. 1, 1874, and paid Aug. 12, 1876. What was the amount due?

19. A man engaged in business was making 12½% annually on his capital of $16840. He quit his business and loaned his money at 7½%. What did he lose in 2 yr. 3 mo. 18 da. by the change?

20. A man borrows $2876.75, which belongs to a minor who is 16 yr. 5 mo. 10 da. old, and he is to retain it until the owner is 21 years old. What will then be due at 8% simple interest?

21. A speculator borrowed $9675, at 6%, April 15, 1874, with which he purchased flour at $6.25. May 10, 1875, he sold the flour at $7⅜ a barrel, cash. What did he gain by the transaction?

22. A man borrows $10000 in Boston at 6%, reckoning 360 da. to the year, and lends it in Ohio at 8%, reckoning 365 da. to the year. What will be his gain in 146 days?

23. A tract of land containing 450 acres was bought at $36 an acre, the money paid for it being loaned at 5½%. At the end of 3 yr. 8 mo. 24 da., ⅔ of the land was sold at $40 an acre, and the remainder at $38½ an acre. What was *gained or lost by* the transaction?

PROBLEMS IN INTEREST.

576. Interest, time, and rate given, to find the principal.

ORAL EXERCISES.

1. What sum of money will gain $10 in 1 yr. at 5%?

ANALYSIS.—The interest of $1 for 1 yr. at 5% is .05 of the principal, and therefore $10 ÷ .05, or $200, is the required sum. Or,

Since $.05 is the interest of $1, $10 is the interest of as many dollars as $.05 is contained times in $10, or 200 times. Hence, etc.

What sum of money will gain,

2. $20 int. in 2 yr. at 5%?
3. $25 int. in 5 yr. at 5%?
4. $60 int. in 2 yr. at 6%?
5. $84 int. in 2 yr. at 7%?
6. $50 int. in 6 mo. at 10%?
7. $30 int. in 3 mo. at 8%?

WRITTEN EXERCISES.

577. 1. What sum of money, put at interest 3½ yr. at 6%, will gain $346.50?

OPERATION.

Int. of $1 for 3½ yr. at 6% = $.21.
$346.50 ÷ $.21 = 1650 times;
$1 × 1650 = $1650.

ANALYSIS.—Same as in oral exercises. (**576.**)

What *principal*

2. Will gain $49.50 in 1 yr. 3 mo. at 6%? At 5%?
3. Will gain $153.75 in 3 mo. 24 da. at 7%? At 8%?

RULE.—*Divide the given interest by the interest of $1 for the given time, at the given rate.*

FORMULA.—*Principal* = *Interest* ÷ (*Rate* × *Time*).

What sum of money

4. *Will gain* $213 in 5 yr. 10 mo. 20 da. at 7%?
5. *Will gain* $173.97 in 4 yr. 4 mo. at 6%? At 12%?

6. A man receives semi-annually $350 int. on a mortgage at 7%. What is the amount of the mortgage?

578. Amount, rate, and time given, to find the principal.

ORAL EXERCISES.

1. What sum of money will amount to $107 in 1 yr. at 7%?

ANALYSIS.—Since the interest is .07 of the principal, the amount is 1.07, or $\frac{107}{100}$, of it. If $107 is $\frac{107}{100}$ of the principal, $\frac{1}{100}$ of the principal is $\frac{1}{107}$ of $107, or $1; and $\frac{100}{100}$, or the principal itself, is $100. Or,

Since $1.07 is the amount of $1, $107 is the amount of as many dollars as $1.07 is contained times in $107, or $100.

What sum of money will amount to

2. $130 in 5 yr. at 6%?
3. $228 in 2 yr. at 7%?
4. $412 in 6 mo. at 6%?
5. $250 in 10 yr. at 10%?
6. $350 in 15 yr. at 5%?
7. $260 in 3 yr. 9 mo. at 8%?

WRITTEN EXERCISES.

579. 1. What sum will amount to $337.50 in 5 yr. at 7%?

OPERATION.

Am't of $1 for 5 yr. at 7% = $1.35.
$337.50 ÷ $1.35 = 250 times;
$1 × 250 = $250.

ANALYSIS.—Same as in oral exercises. (**578.**)

What *principal*

2. Will amount to $1028 in 4 mo. 24 da. at 7%?
3. Will amount to $1596 in 2 yr. 6 mo. at 5½%?
4. Will amount to $1531.50 in 3 mo. 18 da. at 7%?

RULE.—*Divide the given amount by the amount of* $1 *for the given time, at the given rate.*

FORMULA.—*Prin.* = *Amt.* ÷ $(1 + \overline{\text{Rate} \times \text{Time}})$.

5. What is the interest of that sum which in 217 days, at 5½%, amounts to $918.73?

6. What principal in 3 yr. 4 mo. 24 da. will amount to $761.44 at 5%?

580. Principal, interest, and time given, to find the rate.

ORAL EXERCISES.

1. At what rate will $100 gain $14 in 2 years?

Analysis.—Since the interest of $100 is $14 for 2 yr., it is $7 for 1 yr., and $7 is .07 of $100, the principal. Hence the rate is 7%. Or,

Since the interest of $100 for 2 yr. at 1% is $2, $14 is as many per cent. as $2 is contained times in $14, or 7%.

At what rate will

2. $300 gain $60 in 4 yr.?
3. $500 gain $100 in 5 yr.?
4. $400 gain $84 in 3 yr.?
5. $5 gain $1 in 3 yr.?
6. $120 gain $60 in 10 yr.?
7. $150 double itself in 10 yr.?

WRITTEN EXERCISES.

581. 1. At what rate per cent. will $1600 gain $280 interest in 2½ years?

OPERATION.

Int. of $1600 at 1% for 2½ yr. = $40.
$280 ÷ $40 = 7 times; 1% × 7 = 7%.

Analysis.—Same as in oral exercises. (**580.**)

At what *rate per cent*

2. Will $2085 gain $68.11 in 5 mo. 18 da.?
3. Will $1500 gain $252 in 2 yr. 4 mo. 24 da.?

Rule.—*Divide the given interest by the interest of the given principal, for the given time, at 1%.*

Formula.—*Rate = Int. ÷ (Prin. × 1% × Time).*

4. A house that cost $14500 rents for $1189. What per cent. does it pay on the investment?

5. At what rate will $1500 amount to $1684.50 in 2 yr. 18 da.?

6. At what rate per *month* will $2000 gain $120 in 90 da.?

7. A man invests $15600, which gives him an annual income of $1620. What rate of interest does he receive?

8. At what rate per annum will any sum double itself in 4, 6, 8, and 10 years, respectively?

At 1%, any sum will double itself in 100 yr.; hence, to double itself in 4 yr., the rate will be as many times 1% as 4 yr. are contained times in 100 yr., or 25%, etc.

9. At what rate per annum will any sum triple itself in 2, 5, 7, 12, and 20 years, respectively?

10. I invest $49500 in a business that pays me $297 a month. What annual rate of interest do I receive?

11. Which is the better investment, and how much, one of $4200, yielding $168 semi-annually, or one of $7500, producing $712½ annually?

582. Principal, interest, and rate given, to find the time.

ORAL EXERCISES.

1. In what time will $200 gain $56 at 7%?

ANALYSIS.—The given interest, $56, is 56/200, or .28, of the principal, $200; therefore, the time is as many years as .07, the given rate, is contained times in .28, or 4 times. Hence, etc.

Or, the interest of $200 at 7% for 1 yr. is $14; therefore, the time is as many years as $14 are contained times in the given interest, $56, or 4 years. Hence, etc.

In what time will

2. $40 gain $10 at 5%?
3. $500 gain $100 at 4%?
4. $25 gain $20 at 6%?
5. $1000 gain $250 at 5%?
6. $5 gain 90 cents at 6%?
7. $50 gain $12½ at 10%?

WRITTEN EXERCISES.

583. 1. In what time will $840 gain $78.12 at 6%?

OPERATION.

$840 × .06 = $50.40 Int. for 1 yr.
$78.12 ÷ $50.40 = 1.55.
1 yr. × 1.55 = 1 yr. 6 mo. 18 da.

ANALYSIS.—Same as in the oral exercises. (**582.**)

In what time

2. Will $175.12 gain $6.43 at 6%?

3. Will $1000 amount to $1500 at 7½%?

RULE.—*Divide the given interest by the interest of the given principal, at the given rate for* 1 *year.*

FORMULA.—*Time* = *Interest* ÷ (*Prin.* × *Rate*).

4. In what time will $8750 gain $1260 at 2% a month?

5. How long must $1301.64 be on interest to amount to $1522.92 at 5%?

6. How long will it take any sum of money to double itself at 3%, 5%, 6%, 7½%, and 10%, respectively?

At 100%, any sum of money will double itself in 1 year; hence to double itself at 10%, it will require as many years as 10% is contained times in 100%, or 10 yr.

7. How long will it take any sum to triple itself at 4%, 5%, 7%, 8%, and 12½%, respectively?

8. In what time will the interest of $120, at 8%, equal the principal? Equal *half* the principal? Equal *twice* the principal?

COMPOUND INTEREST.

584. ***Compound Interest*** is interest not only on the principal, but on the interest added to the principal when it becomes due?

ORAL EXERCISES.

585. 1. What is the comp. int. of \$500 in 2 yr. at 6%?

ANALYSIS.—The simple interest of \$500 for 2 yr. is \$60; the interest of the first year's interest, \$30, for the second year is \$1.80, which, added to \$60, gives \$61.80, the compound interest. Or;

The interest of \$500 for 1 yr. at 6% is \$30, and the amount is \$530, which is the principal for the second year; the interest of \$530 for 1 yr. at 6% is \$31.80, which added to \$530 gives \$561.80, the final amount; and deducting \$500, the original principal, gives \$61.80, the compound interest.

What is the *compound interest*

2. Of \$600 for 2 yr. at 5%?
3. Of \$100 for 2 yr. at 7%?
4. Of \$300 for 2 yr. at 10%?
5. Of \$1000 for 2 yr. at 5%?

What is the *amount* at compound interest,

6. Of \$800 for 2 yr. at 5%?
7. Of \$2000 for 2 yr. at 10%?
8. Of \$400 for 2 yr. at 4%?
9. Of \$500 for 2 yr. at 8%?

WRITTEN EXAMPLES.

586. 1. What is the comp. int. of \$750 for 2 yr. at 6%?

OPERATION.	
\$750	Prin. for 1st yr.
1.06	
\$795	Prin. for 2d yr.
1.06	
\$842.70	Total amount.
750.	
\$92.70	Compound int.

ANALYSIS.—Since the amount is 1.06 of the principal, the amount at the end of the first year is \$795, which is the principal for the 2d year, and the amount at the end of the 2d year is \$842.70. Hence, by subtracting the given principal, \$750, the result is the compound interest, \$92.70.

2. What will $350 amt. to in 3 yr. at 7%, comp. int.?

3. What is the compound int. of $1200 for 3 yr. at 5%?

RULE.—I. *Find the amount of the given principal for the first period of time at the end of which interest is due, and make it the principal for the second period.*

II. *Find the amount of this principal for the next period; and so continue till the end of the given time.*

III. *Subtract the given principal from the last amount, and the remainder will be the compound interest.*

When the time contains months and days, less than a single period, find the amount up to the end of the last period, and compute the simple interest upon that amount for the remaining months and days, which add to find the total amount.

4. What will $864.50 amount to in 4 yr. at 8%, compound interest?

5. What is the compound interest of $680 for 2 yr. at 7%, interest being payable semi-annually?

6. What is the compound interest of $460 for 1 yr. 5 mo. 18 da. at 6%, interest payable quarterly?

7. What will be the amount of $1250 in 3 yr. 7 mo. 18 da. at 5%, interest being semi-annual?

8. Find the compound interest of $790 for 9 mo. 27 da. at 8%, payable quarterly.

The computation of compound interest may be abridged by using the following table.

To use the table, multiply the given principal by the number in the table corresponding to the given number of years and the given rate. If the interest is not annual, reduce the time to periods, and the rate proportionally. Thus, 2 yr. 6 mo., by semi-annual payments, at 7%, is the same as 5 yr. at 3½%; and 1 yr. 9 mo., quarterly payments, at 8%, the same as 7 yr. at 2%.

587. TABLE *showing the amt. of* $1, *at* 2½, 3, 3½, 4, 5, 6, 7, 8, 9, 10, 11, *and* 12%, *compound int., from* 1 *to* 20 *years.*

Yrs.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	6 per cent.
1	1.025000	1.030000	1.035000	1.040000	1.050000	1.060000
2	1.050625	1.060900	1.071225	1.081600	1.102500	1.123600
3	1.076891	1.092727	1.108718	1.124864	1.157625	1.191016
4	1.103813	1.125509	1.147523	1.169859	1.215506	1.262477
5	1.131408	1.159274	1.187686	1.216653	1.276282	1.338226
6	1.159693	1.194052	1.229255	1.265319	1.340096	1.418519
7	1.188686	1.229874	1.272279	1.315932	1.407100	1.503630
8	1.218403	1.266770	1.316809	1.368569	1.477455	1.593848
9	1.248863	1.304773	1.362897	1.423312	1.551328	1.689479
10	1.280085	1.343916	1.410599	1.480244	1.628885	1.790848
11	1.312087	1.384234	1.459970	1.539454	1.710339	1.898299
12	1.344889	1.425761	1.511069	1.601032	1.795856	2.012197
13	1.378511	1.468534	1.563956	1.665074	1.885649	2.132928
14	1.412974	1.512590	1.618695	1.731676	1.979932	2.260904
15	1.448298	1.557967	1.675349	1.800944	2.078928	2.396558
16	1.484506	1.604706	1.733986	1.872981	2.182875	2.540352
17	1.521618	1.652848	1.794676	1.947901	2.292018	2.692773
18	1.559659	1.702433	1.857489	2.025817	2.406619	2.854339
19	1.598650	1.753506	1.922501	2.106849	2.526950	3.025600
20	1.638616	1.806111	1.989789	2.191123	2.653298	3.207136

Yrs.	7 per cent.	8 per cent.	9 per cent.	10 per cent.	11 per cent.	12 per cent.
1	1.070000	1.080000	1.090000	1.100000	1.110000	1.120000
2	1.144900	1.166400	1.188100	1.210000	1.232100	1.254400
3	1.225043	1.269712	1.295029	1.331000	1.367631	1.404908
4	1.310796	1.360489	1.411582	1.464100	1.518070	1.573519
5	1.402552	1.469328	1.538624	1.610510	1.685058	1.762342
6	1.500730	1.586874	1.677100	1.771561	1.870414	1.973822
7	1.605781	1.713824	1.828039	1.948717	2.076160	2.210681
8	1.718186	1.850930	1.992563	2.143589	2.304537	2.475963
9	1.838459	1.999005	2.171893	2.357948	2.558036	2.773078
10	1.967151	2.158925	2.367364	2.593742	2.839420	3.105848
11	2.104852	2.331639	2.580426	2.853117	3.151757	3.478549
12	2.252192	2.518170	2.812665	3.138428	3.498450	3.895975
13	2.409845	2.719624	3.065805	3.452271	3.883279	4.363492
14	2.578534	2.937194	3.341727	3.797498	4.310440	4.887111
15	2.759031	3.172169	3.642482	4.177248	4.784588	5.473565
16	2.952164	3.425943	3.970306	4.594973	5.310893	6.130392
17	3.158815	3.700018	4.327633	5.054470	5.895091	6.866040
18	3.379932	3.996019	4.717120	5.559917	6.543551	7.689964
19	3.616527	4.315701	5.141661	6.115909	7.263342	8.612760
20	3.869684	4.660957	5.604411	6.727500	8.062309	9.646291

9. Find by the table the compound interest of $950 for 1 yr. 5 mo. 24 da., at 10%, interest payable quarterly.

OPERATION.

1 yr. 5 mo. 24 da. = 5 quarters of a year + 2 mo. 24 da.
10% per annum = 2½% per quarter.
Amount for 5 yr. at 2½% = 1.131408 of principal.
$950 × 1.131408 = $1074.837, amount for 1 yr. 3 mo.
Interest of $1074.837 for 2 mo. 24 da. at 10% = $25.079.
$1074.837 + $25.079 = $1099.916, total amount.
$1099.916 − $950 = $149.916, compound interest.

10. Find the amount, at compound interest, of $749.25 for 10 yr. 4 mo., at 7%, interest payable semi-annually.

11. What sum placed at simple interest for 3 yr. 10 mo. 18 da., at 7%, will amount to the same as $1500 placed at compound interest for the same time, and at the same rate, payable semi-annually?

12. At 8%, interest compounded quarterly, how much will $850 amount to in 1 yr. 10 mo. 20 da.?

13. What will $500 amount to in 20 yr. at 7%, comp. int.?

14. A father at his death left $12500 for the benefit of his only son, 14 yr. 8 mo. 12 da. old, the money to be paid him when he should be 21 years of age, with 6% interest compounded semi-annually. What did he receive?

ANNUAL INTEREST.

588. ***Annual Interest*** is interest on the principal and on each year's interest remaining unpaid, but so computed as not to increase the original principal.

It is allowed in the case of promissory notes and other contracts which contain the words, "with interest payable annually," or with "compound interest." In such cases, the interest is not compounded *beyond* the second year.

WRITTEN EXERCISES.

589. 1. Find the annual interest and amount of \$8000 for 5 yr., at 6% per annum.

OPERATION.

Int. of \$8000 for 5 yr. at 6% = \$2400.
" " \$480 for 10 yr. at 6% = \$288.
\$2400 + \$288 = \$2688, Annual int.
\$8000 + \$2688 = \$10688, Amount.

ANALYSIS.—The interest on \$8000 for 1 yr. at 6% is \$480, and for 5 yr. is \$2400.

The interest for the first year, remaining unpaid, draws interest for 4 yr.; that for the second year, for 3 yr.; that for the third year, for 2 yr.; and that for the fourth year, for 1 yr., the sum of which is equal to the interest of \$480 for 4 yr. + 3 yr. + 2 yr. + 1 yr. = 10 yr.; and the interest of \$480 at 6% for 10 yr. is \$288. Hence the total amount of interest is \$2400 + \$288, or \$2688, and the amt. is \$10688.

2. What is the annual interest of \$1500 for 4 yr. at 7%?

RULE.—I. *Compute the interest on the principal for the given time and rate, to which add the interest on each year's interest for the time it has remained unpaid.* Or,

II. *When the interest has remained unpaid for a number of years, multiply the interest for one year by the product of the number of years and half that number diminished by one.*

Thus, if the time is 9 yr., the interest for 1 yr. should be multiplied by $9 \times (9-1) \div 2$, or $9 \times 4 = 36$. Since the interest for the first year draws 8 years' interest, that for the second year 7 years' interest, etc., and the sum of the series $8+7+6+5+4+3+2+1$ is 36.

3. What will \$3500 amt. to in 10 yr., annual int., at 8%?

4. What is the difference between the annual interest and the compound interest of \$2500 for 6 yr. at 6%?

5. Find the amt. of \$575, at 8% annual int., for 9½ yr.

6. $800. MACON, June 15, 1872.

Four years after date, for value received, I promise to pay Robert E. Park, or order, eight hundred dollars, with interest at seven per cent., payable annually.

J. W. BURKE.

What amount is due on this note at maturity, no interest having been paid?

PARTIAL PAYMENTS.

590. ***Partial Payments*** are payments in part of the amount of a note, bond, or other obligation.

591. ***Indorsements*** are the acknowledgment of such payments, written on the back of the note, bond, etc., stating the time and amount of the same.

592. A ***Promissory Note*** is a written promise to pay a certain sum of money, on demand or at a specified time.

593. The ***Maker or Drawer*** of the note is the person who signs it.

594. The ***Payee*** is the person to whom, or to whose order, the money is paid.

595. An ***Indorser*** is a person who, by signing his name on the back of the note, makes himself responsible for its payment.

596. The ***Face*** of a note is the sum of money made payable by the note.

597. A ***Negotiable Note*** is one made payable to bearer, or to any person's order. When so made, it can be sold or transferred.

WRITTEN EXERCISES.

1. $800. NEW YORK, Jan. 1st, 1874.

One year after date, I promise to pay Caleb Barlow, or order, eight hundred dollars, for value received, with interest. JAMES DUNLAP.

Indorsed as follows: April 1, 1874, $10; July 1, 1874, $35; Nov. 1, 1874, $100. What was there due Jan. 1, 1875?

ANALYSIS.—The interest of $800 for 3 mo., from Jan. 1 to April 1, at 7%, is $14; am't, $814. Since the payment is less than the interest, it cannot be deducted for a new principal without compounding the int., which is illegal; hence, find the int. of $800 to the time of the next payment, 3 mo., which is $14, and the amt. to that time, $828, from which deduct the sum of the two payments, or $45, leaving $783, a new principal. The int. of $783 for 4 mo., to Nov. 1, is $18.27; amt., $801.27; from which deduct the third payment, $100, leaving $701.27, the next principal, the amt. of which for 2 mo., to Jan. 1, 1874, is $709.45, sum due at that time.

PRINCIPLE.—*The principal must not be increased by the addition of interest due at the time of any payment, so as to compound the interest.*

Upon this principle is based the rule prescribed by the Supreme Court of the United States:

U. S. RULE.—I. *Find the amount of the given principal to the time of the first payment, and if this payment equals or exceeds the interest then due, subtract it from the amt. obtained, and treat the remainder as a new principal.*

II. *If the interest exceed the payment, find the amount of the same principal to a time when the sum of the payments equals or exceeds the interest then due, and subtract the sum of the payments from that amount.*

III. *Proceed in the same manner with the remaining payments.*

$500. PHILADELPHIA, Feb. 1, 1875.

2. *Three months after date, I promise to pay to J. B Lippincott & Co., or order, five hundred dollars, with interest, without defalcation. Value received.*

JAMES MONROE.

Indorsed as follows: May 1, 1875, $40; Nov. 14, 1875, $8; April 1, 1876, $18; May 1, 1876, $30. What was due Sept. 16, 1876?

OPERATION.

Face of note, or principal		$500.00
Interest to May 1, 1875, 3 mo., at 6%		7.50
Amount		507.50
Payment, to be subtracted		40.00
2d principal		467.50
Int. of $467.50 to Nov. 14, 1875, 6 mo. 13 da.	$15.04	
Int. of $467.50 to April 1, 1876, 4 mo. 17 da.	10.67	25.71
Amount		493.21
Sum of payments, to be subtracted		26.00
3d principal		467.21
Int. to May 1, 1876, 1 mo.		2.34
Amount		469.55
Payment, to be subtracted		30.00
4th principal		439.55
Int. to Sept. 16, 1876, 4 mo. 15 da.,		9.89
Amount due		$449.44

3. What was the amount due October 25, 1873, upon a note for $1500, dated New Orleans, April 1, 1872, and on which the following payments were endorsed: June 5, 1872, $300; Oct. 15, 1872, $37.75; May 1, 1873, $97.25; Aug. 6, 1873, $495?

$700. DETROIT, Nov. 1, 1873.

4. *On demand, I promise to pay Charles Smith, or order, seven hundred dollars, with interest. Value received.*

ABRAHAM ISAACS.

Indorsed as follows: Dec. 5, 1873, $75; Jan. 10, 1874, $350; April 11, 1874, $11.25; May 15, 1874, $250. What was due Sept. 1, 1874?

497\frac{39}{100}$. CHICAGO, March 15, 1874.

5. *Three months after date, we promise to pay James Kelly, or order, four hundred and ninety-seven $\frac{39}{100}$ dollars, with interest. Value received.*

BROWN, NICHOLS & CO.

Indorsed as follows: Nov. 3, 1874, $57.50; June 15, 1875, $22.25; Aug. 1, 1875, $125; Sept. 15, 1875, $175. What was due Jan. 1, 1876?

598. The following method of computation is often used by merchants in the settlement of notes and of interest accounts running a year or less; hence called the MERCANTILE RULE:

I. *Find the amount of the note or debt from its date to the time of settlement.*

II. *Find the amount of each payment from its date to the time of settlement.*

III. *Subtract the sum of the amounts of payments from the amount of the note or debt.*

An accurate application of this rule requires that the time should be reduced to *days*, and that the interest should be computed by the rule for days (**574**).

1. On a note for $600 at 7%, dated Feb. 15, 1874, were the following indorsements: March 25, 1874, $150; June 1, 1874, $75; Oct. 10, 1874, $100. What was due Dec. 31, 1874?

OPERATION.

Am't of $600 from Feb. 15 to Dec. 31,	319 da.,		$636.71
" " $150 " Mar. 25 " "	281 da.,	$158.08	
" " $75 " June 1 " "	213 da.,	78.06	
" " $100 " Oct. 10 " "	82 da.,	101.57	337.71
Balance due Dec. 31, 1874,			$299.00

2. A note for $950, dated Jan. 25, 1876, payable in 9 mo., at 7% interest, had the following indorsements: March 2, 1876, $225; May 5, 1876, $174.19; June 29, 1876, $187.50; Aug. 1, 1876, $79.15. What was the balance due at the time of its maturity?

3. Payments were made on a debt of $1750, due April 5, 1875, as follows: May 10, 1875, $190; July 1, 1875, $230; Aug. 5, 1875, $645; Oct. 1, 1875, $372. What was due Dec. 31, 1875, interest at 6%?

DISCOUNT.

599. ***Discount*** is a certain per cent deducted from the price-list of goods, or an allowance made for the payment of a debt or other obligation before it is due.

600. The ***Present Worth*** of a debt payable at a future time without interest, is such a sum as, being put at legal interest, will amount to the debt when it becomes due.

601. The ***True Discount*** is the difference between the whole debt and the present worth.

ORAL EXERCISES.

602. 1. What is the present worth of a debt of $224, to be paid in 2 yr., at 6%?

ANALYSIS.—Since in 2 yr., at 6%, the int. is .12 of the principal, the amt. is 1.12 of it; therefore, $224, the debt, is 1.12, or $\frac{112}{100}$ of the present worth, and $\frac{100}{100}$, or the present worth itself, is $200. Or, since $1.12 is the amt. of $1, $224 is the amt. of as many dollars as $1.12 is contained times in $224, or $200. (**578.**)

What is the present worth

2. Of $315, due in 10 mo., at 6%?
3. Of $570, due in 2 yr., at 7%?
4. Of $408, due in 3 mo., at 8%?
5. Of $51, due in 4 mo., at 6%?
6. Of $440, due in 2 yr., at 5%?

Find the true discount at 6%,

7. Of $1019, due in 3 mo. 24 da.
8. Of $102.20, due in 4 mo. 12 da.
9. Of $5035, due in 1 mo. 12 da.

WRITTEN EXERCISES.

603. 1. What are the present worth and the true discount, of $362.95, payable in 7 mo. 12 da., at 6%?

OPERATION.

Amt. of $1, for 7 mo. 12 da., at 6% = $1.037
$362.95 ÷ $1.037 = 350 times.
$1 × 350 = $350, Present Worth.
$362.95 − $350 = $12.95, True Discount.

ANALYSIS.—Since the amount of $1 for 7 mo. 12 da. at 6% is $1.037 (**579**), $362.95 is the amount of as many dollars as $1.037 is contained times in $362.95, or 350 times. Hence the *present worth* is $350; and the *true discount* is $362.95 − $350, or $12,95.

2. What is the present worth of a debt of $287.75 to be paid in 3 mo. 18 da. at 7%?

3. What is the true discount on a debt of $2202.90 due in 8 mo. 12 da. at 7%?

RULE.—I. *Divide the debt by the amount of* $1 *for the given rate and time, and the quotient is the present worth.*

II. *Subtract the present worth from the debt, and the remainder is the true discount.*

FORMULA.—*Present Worth* = *Debt* ÷ *Amt. of* $1.

Hence the present worth is the *principal* of which the true discount is the *interest*, and the whole debt the *amount*.

4. Bought a house and lot for $19500 cash, and sold them for $22000, payable one-fourth in cash and the remainder in 1 yr. 6 mo. How much ready money did I gain, computing discount at 6%?

5. A merchant buys goods for $4200 on 4 mo. credit, but is offered a discount of 3% for cash. If money is worth ½% a month, what is the difference?

6. Bought a bill of lumber amounting to $3500, on 6 mo. credit; 2 months afterward paid on account $1500, and 1 month later, $1000. Find the present worth of the balance, at the time of the second payment, int. at 7%.

7. A merchant holds two notes, one for $356.25 due Dec. 1, 1875, and the other for $497.50, due Feb. 1, 1876. What would be due him in cash on both notes Sept. 15, 1875, at 6%?

8. A bookseller bought $300 worth of books at a discount of 33⅓% from list prices, and sold them at the regular retail price, on 6 mo. time. Money being worth 6%, what per cent. profit did he make?

9. A speculator bought 230 bales of cotton, each bale containing 470 lb., at $11\frac{3}{4}$ cents a pound, on a credit of 9 mo. He at once sold the cotton for $13000 cash, and paid the pres. worth of the debt at 7%. What was his gain?

10. Which is the more profitable, to buy flour at $8.75 a barrel on 6 mo. credit, or at $8.60 on 2 mo., money being worth 7%?

11. A person sold goods to the amount of $3750, 15% payable in cash, 25% in 3 mo., 20% in 4 mo., and the remainder in 6 mo. What ready money would discharge the whole debt, money being worth 6%?

BANK DISCOUNT.

604. A ***Bank*** is a corporation chartered by law for the safe-keeping and loaning of money, or the issuing of bills for circulation as money.

605. ***Bank Bills*** or ***Notes*** are promissory notes issued by banks, and payable on demand.

A bank which issues notes to circulate as money is called a *Bank of Issue;* one which lends money by discounting notes, a *Bank of Discount;* and one which takes charge of money belonging to other parties, called *depositors,* a *Savings Bank,* or *Bank of Deposit.* Some banks perform two and others all of these duties.

606. ***Bank Discount*** is a deduction made by a bank for paying a note before it is due. This deduction is the *interest* on the face of the note for the time it has to run, including the days of grace. It is, therefore, interest paid *in advance.*

607. ***Days of Grace*** are the three days usually allowed by law for the payment of a note after the expiration of the time specified in the note.

608. The ***Maturity*** of a note is the expiration of the whole time, including the days of grace.

609. The ***Term of Discount*** is the time from the discount of a note to its maturity.

610. A ***Bank Check*** is a written order for money by a depositor, upon a bank.

611. The ***Proceeds*** or ***Avails*** of a note is the sum received for it when discounted, that is, the face of the note less the discount.

612. A ***Protest*** is a formal declaration in writing, made by a Notary-Public, at the request of the holder of a note, to give legal notice to the maker and the indorsers of its non-payment.

1. The failure to protest a note on the third day of grace releases the indorsers from all obligation to pay it.

2. If the third day of grace or the maturity of a note occurs on Sunday or a legal holiday, it must be paid on the day previous.

3. The transaction of borrowing money at a bank is conducted as follows: The borrower presents a note, either made or indorsed by himself, payable at a specified time, and receives for it a sum equal to the face less the interest for the time it has to run, including the days of grace. A note for discount at a bank must be made payable to the order of some person, by whom it must be indorsed. When the note bears interest, the discount is computed on its face plus the interest for the time it has to run.

613. Bank discount being simple interest, the following are corresponding terms:

The ***Face of the Note*** is the *principal.*
The ***Term of Discount*** is the *time.*
The ***Bank Discount*** is the *interest.*
The ***Proceeds*** is the *principal less the interest.*

614. To find the bank discount and proceeds of a note.

ORAL EXERCISES.

1. What is the bank discount on a note for $2000 due in 2 mo. 15 da. at 6%, and the proceeds?

ANALYSIS.—After adding 3 da., the time is 2 mo. 18 da.; the interest for which at 6% is .013 of the principal; .013 of $2000 is $26, the *bank discount*, and $2000 − $26 equals $1974, or the *proceeds*.

What are the *bank discount* and the *proceeds* of a note

2. Of $80 for 5 mo. 27 da., at 7%?
3. Of $100 for 2 mo. 21 da., at 6%?
4. Of $200 for 8 mo. 9 da., at 7%?
5. Of $150 for 4 mo. 21 da., at 5%?
6. Of $100 for 30 da., at 6%?

WRITTEN EXERCISES.

615. 1. Required the bank discount and proceeds of a note for $1250 due in 90 days, at 7%.

OPERATION.

$1250 × .0155 = $19.375
$19.374 ÷ 6 = 3.229
$22.604 Bank Disc't.
$1250 − $22.604 = $1227.396. Proceeds.

ANALYSIS.—The interest of $1250 for 93 da. at 6% is .0155 of it, or $19.375; and 7% of it is $\frac{1}{6}$ more, or $22.604, which is the *bank discount*. Deducting this from $1250, the face of the note, the remainder $1227.396 is the *proceeds*.

RULE.—I. *Compute the interest on the face of the note (or if it bears interest, on its amount at maturity), for three days more than the specified time, and the result is the bank discount.*

II. *Subtract the discount from the face of the note, or its amount at maturity, and the remainder is the proceeds.*

2. What is the bank discount, and what is the proceeds of a note for \$597.50, due in 60 da., at 6%?

3. What will be the proceeds of a note for \$1615, due in 90 da. with interest at 7%, discounted at the Nassau Bank in New York?

4. Sold a farm, containing 173 A. 95 P., for \$62½ an acre, and received payment as follows: \$2000 cash, and the balance in a note payable in 5 mo. 18 da. at 7% interest, which was discounted at a bank. How much ready money did the farm bring?

Find the *date of maturity*, the *term of discount*, and the *proceeds* of the following:

$\$957\frac{37}{100}$. CHICAGO, July 27, 1875.

5. Three months after date, I promise to pay to the order of D. L. Moody, nine hundred fifty-seven and $\frac{37}{100}$ dollars, for value received.

Discounted Aug. 10, at 8%. WILLIAM THOMSON.

$\$916\frac{25}{100}$. SAN FRANCISCO, Feb. 5, 1874.

6. Two months after date, we jointly and severally agree to pay C. H. Thomas, or order, nine hundred sixteen and $\frac{25}{100}$ dollars *with interest* at 8%, value received.

Discounted at Marine Bank, JAMES BARNES.
Feb. 21, at 10%. GEORGE CHILDS.

$\$1315\frac{75}{100}$. NEW YORK, May 1, 1875.

7. Ninety days after date, I promise to pay to the order of Ivison, Blakeman, Taylor & Co., one thousand three hundred fifteen and $\frac{75}{100}$ dollars, for value received.

Discounted at WILLIAM HEWSON.
Broadway Bank, May 15, at 7%.

$1250. BOSTON, June 12, 1876.

8. Six months after date, I promise to pay Knight, Adams & Co., or order, twelve hundred fifty dollars, *with interest* at 6 per cent., value received.

Discounted at a broker's, GEO. B. DAMON.
Nov. 15, at 6%.

616. The proceeds and time of a note given, to find the face.

ORAL EXERCISES.

1. For what sum must a note be drawn, at 2 mo. 15 da., at 6%, so that the proceeds when discounted may be $987?

ANALYSIS.—The bank discount for 2 mo. 18 da. at 6% is .013 of the face of the note, and the proceeds must therefore be 1 − .013, or .987 of the face; and if .987 of the face is $987, the whole face of the note is $1000.

Required the face of a note, so that the proceeds may be

2. $972, for 4 mo. 21 da. at 7%.
3. $194, for 5 mo. 27 da. at 6%.
4. $97.60, for 3 mo. 15 da. at 8%.
5. $980, for 4 mo. 21 da. at 5%.
6. $184, for 9 mo. 15 da. at 10%.

WRITTEN EXERCISES.

617. 1. What must be the face of a note at 9 mo. 27 da., interest 8%, so that the proceeds may be $448?

OPERATION.

The bank discount of \$1 for 10 mo. at 8% is $\$.066\frac{2}{3}$.
The proceeds of \$1 = \$1 − $\$.066\frac{2}{3}$ or $\$.933\frac{1}{3}$.
Hence \$448 ÷ $.933\frac{1}{3}$ = \$480, the face of the note.

2. What is the face of a note at 30 da., the proceeds of which, when discounted at bank, at 7%, are $1425?

RULE.—*Divide the given proceeds by the proceeds of $1 for the time and rate given; the quotient is the face of the note.*

FORMULA.—$\textit{Face} = \textit{Proceeds} \div (1 - \overline{\textit{Rate} \times \textit{Time}})$.

3. Find the face of a 3 mo. note the proceeds of which, discounted at 2% a month, is $675.

4. The proceeds of a note are $1915.75, the time 3 mo., and the rate of interest 7%; what is the face of the note?

5. Bought merchandise for $2250, cash; for what sum must I draw my note at 3 mo., so as to obtain that sum at the bank, interest at 7%?

6. The avails of a 3 months note, when discounted at 7½%, were $315.23; what was the face of the note?

7. For what sum must a note dated April 5, for 90 da., be drawn, so that when discounted at 7%, on April 21, the proceeds may be $650?

8. For how much must I draw my note at 90 da., in order that when discounted at a bank, at 7%, its avails will pay for 137¾ yd. of cloth at $2⅝ a yard?

SAVINGS-BANK ACCOUNTS.

618. A ***Savings-Bank*** is designed chiefly to accommodate depositors of small sums of money.

Interest is allowed semi-annually on all sums that have been on deposit for a certain time, if not drawn out before the regular day of paying interest—generally on the 1st of January and of July.

Savings-banks generally allow interest only from the commencement of each quarter; but in some banks money deposited previous to the 1st day of any month draws interest from that date to the day of declaring interest dividends, provided it has not been previously withdrawn.

WRITTEN EXERCISES.

619. 1. A person had on deposit Jan. 1, 1874, $150. His subsequent deposits were, Feb. 3, $35; March 29, $20; April 10, $43; May 15, $26. His drafts during the same time were, Jan. 15, $50; Feb. 27, $15; April 19, $45. What interest was due July 1st, at 6%?

OPERATION.

Date of Int. paym'ts.	Balance 1st of month.	Smallest Bal. during mo.	Interest for 1 month.	Smallest Bal. dur'g Q'rter.	Interest for 1 Quarter.
JAN. 1	$150				
Feb. 1	100	$100	$.50		
Mar. 1	120	100	.50		
APR. 1	140	120	.60	$100	$1.50
May 1	138	138	.69		
June 1	164	138	.69		
JULY 1	164	164	.82	138	2.07
			$3.80		$3.57

Balance due, with int. by *monthly* periods, $167.80.
" " " " " *quarterly* " $167.57.

ANALYSIS.—At the end of January, the balance due is $100, which having been on deposit for the month, draws interest for 1 mo.; at the end of February, the balance is $120; but the *smallest balance* during the month is $100; hence interest is allowed only on that sum. The same principle applies to the other balances. If only *quarterly periods* of interest are allowed, the interest is calculated at the end of each quarter on the smallest balance during the quarter, or, in this case, on $100, April 1, and $138, July 1.

2. Find the balance, due July 1, on the following account: *Deposits,* Jan. 15, $175; April 10, $60; May 31, $110. *Drafts,* March 5, $75; May 1, $35; June 10, $50. Interest at 6%, from the 1st day of each month.

3. A person deposits in a savings-bank the following sums: Jan. 1, $350; Feb. 5, $150; March 15, $75; May 10, $30; June 15, $100. During the same time he draws, Jan. 15, $150; Feb. 10, $200; March 31, $50; June 1, $75. What interest at 6%, payable from the 1st of each month, must be added to the account July 1?

4. Balance the following, Jan. 1, 1875: Balance due to Margaret Brown, July 1, 1874, $275. Deposits received as follows: Aug. 1, $125; Sept. 15, $57; Oct. 10, $350. Drafts paid: July 15, $100; Sept. 1, $150; Nov. 15, $68; Dec. 15, $125. Interest at 6%, from the 1st of each quarter, July 1 and Oct. 1.

RULE.—*At the end of each term compute the interest for the term on the smallest balance on deposit at any time during the quarter; and at the end of each period of six months add to the balance of principal the whole amount of interest due, and the sum will be the principal at the commencement of the next six months.*

5. How much was due Jan. 1, 1876, on the following account, allowing interest, computed from the 1st of each quarter, Jan. 1 to July 1, at 6% per annum?

Dr. GREENWICH SAVINGS BANK, *in acct. with* MARY WILLIAMS. *Cr.*

1874.				1874.			
Jan. 1	To Cash .	$136	00	Sept. 15	By Check	$75	00
Mar. 17	" " .	25	00	1875.			
Aug. 1	" " .	87	50	Jan. 20	" "	37	50
1875.				Mar. 3	" "	50	00
June 11	" " .	150	00				
Nov. 17	" " .	72	00				

620. SYNOPSIS FOR REVIEW.

PERCENTAGE—CONTINUED.

- 10. INTEREST.
  - 1. Defs.
    - 1. *Interest.* 2. *Principal.* 3. *Rate.* 4. *Amt.* 5. *Legal Int.* 6. *Usury.*
  - 2. Corresponding Elements.
  - 3. 1. Principle. 2. Rule, I, II, III.
  - 4. Relations between Time and Interest. I, II, III, IV.
  - 5. **569.** Rule, I, II, III, IV.
  - 6. 6% Method.
    - 1. Principles, 1, 2.
    - 2. Rule.
  - 7. Accurate Interest. Rule.
  - 8. Problems.
    - **576.** 1. *Rule.* 2. *Formula.*
    - **578.** 1. *Rule.* 2. *Formula.*
    - **580.** 1. *Rule.* 2. *Formula.*
    - **582.** 1. *Rule.* 2. *Formula.*
- 11. COMPOUND INTEREST.
  - 1. Definitions—Compound Interest.
  - 2. Rule, I, II, III.
- 12. ANNUAL INTEREST.
  - 1. Definitions.
  - 2. Rule, I, II.
- 13. PARTIAL PAYMENTS.
  - 1. Defs.
    - 1. *Part. Pay'ts.* 2. *Indorsem'ts.* 3. *Promissory Note.* 4. *Maker or Drawer.* 5. *Payee.* 6. *Indorser.* 7. *Face of a Note.* 8. *Negotiable Note.*
  - 2. Principle. U. S. Rule, I. II. Merc. Rule.
- 14. DISCOUNT.
  - 1. Defs.
    - 1. *Discount.* 2. *Present Worth.* 3. *True Discount.*
  - 2. Rule, I, II.
- 15. BANK DISCOUNT.
  - 1. Defs.
    - 1. *Bank.* 2. *Bank Bills or Notes.* 3. *Bank Discount.* 4. *Days of Grace.* 5. *Maturity of Note.* 6. *Term of Discount.* 7. *Bank Check.* 8. *Proceeds or Avails,* 9. *Protest.*
  - 2. Corresponding Terms.
  - 3. **614.** Rule, I, II.
  - 4. **616.** Rule
- 16. SAVINGS-BANK ACCOUNTS—Rule.

STOCKS

621. A ***Corporation*** is an association of individuals authorized by law to transact business as a single person.

622. A ***Charter*** is the legal act of incorporation defining the powers and obligations of the body incorporated.

623. The ***Capital Stock*** of a corporation is the capital or money contributed and employed to carry on the business of the company.

624. ***Certificates of Stock*** or ***Scrip*** are the papers or documents issued by a corporation, specifying the number of shares of the joint capital to which the holders are entitled.

625. A ***Share*** is one of the equal parts into which capital stock is divided.

The value of a share in the original contribution of capital varies in different companies. In bank, insurance, and railroad companies, it is usually $100.

626. ***Stocks*** is a general term applied to shares of stock of various kinds, Government and State bonds, etc.

Stockholders are the owners of stock, either by original title or by subsequent purchase. The stockholders constitute the company.

627. The ***Par Value*** of stock is the sum for which the scrip or certificate was issued.

628. The ***Market Value*** of stock is the sum for which it can be sold.

Stock is *at par* when it can be sold for its original or face value, or 100%; it is *above par*, or *at a premium*, when it will bring more than its face value; and it is *below par*, or *at a discount*, when it sells for less than its face value. Thus, when stock is at par, it is quoted at 100; when it is 5% above par, at 105; and when it is 5% below par, at 95.

629. ***Premium, Discount,*** and ***Brokerage*** are each a *percentage* computed upon the par value of the stock as the base.

630. A ***Stock Broker*** is a person who buys and sells stocks, either for himself, or as the agent of another.

631. ***Stock-jobbing*** is the buying and selling of stocks with the view to realize gain from their rise and fall in the market.

632. An ***Installment*** is a portion of the capital stock required of the stockholders as a payment on their subscription.

633. An ***Assessment*** is a sum required of stockholders, to meet the losses or to pay the business expenses of the company.

634. A ***Dividend*** is a sum paid to the stockholders from the profits of the business.

Dividends and assessments are a *percentage* computed upon the par value of the stock as the *base*.

635. ***Net Earnings*** are the moneys left after paying expenses, losses, and the interest upon the bonds, if there are any.

636. A ***Bond*** is a written instrument securing the payment of a sum of money at or before a specified time.

The principal bonds dealt in by brokers are Government, State, City, and Railroad bonds.

637. *U. S. Bonds* are of two kinds; viz., those which are payable at a fixed date, and those which, while payable at a fixed date, may be paid at an earlier specified time, as the Government may elect.

1. The former are quoted in commercial transactions by the rate of interest which they bear; thus, United States bonds bearing 6% interest are quoted *U. S. 6's.* The latter are quoted in commercial transactions by a combination of the two dates; thus, *U. S. 5-20's*, or *U. S. 6's 5-20*, means bonds of U. S. bearing 6% interest, and payable in 5 or 20 years, as the Government may choose.

2. When it is necessary to distinguish different issues bearing the same rate of interest, the year at which they become due is also mentioned; thus, *U. S. 5's of '71; U. S. 5's of '74; U. S. 6's, 5-20, of '84; U. S. 6's, 5-20, of '85.*

3. The 5-20's were issued in 1862, '64, '65, '67, and '70. They bear interest at 6%, paid semi-annually in gold, except the issue of 1870, called 5's of '81, which bear int. at 5%, paid quarterly in gold.

4. Bonds issued by States, cities, etc., are quoted in a similar manner. Thus, *S. C. 6's* are bonds bearing 6% interest, issued by the State of South Carolina.

638. A ***Coupon*** is a certificate of interest attached to a bond, to be cut off and presented for payment when the interest is due.

639. *Currency* is a term used to denote the circulating medium employed as a substitute for gold and silver. It consists, at present, in the United States, of U. S. Legal-tender Notes, or "Greenbacks," and the Bills issued by the Nat. Banks, and secured by U. S. Bonds.

If from any cause the paper medium depreciates in value, gold becomes an object of investment, the same as stocks. Gold being of fixed standard value, its fluctuations in price indicate changes in the value of the currency. Hence, when gold is said to be at a *premium*, currency is virtually below par, or at a *discount*.

ORAL EXERCISES.

640. 1. Find the cost of 100 shares of Chicago and Rock Island Railroad stock at 90; brokerage ⅛%.

ANALYSIS.—Since the cost of one share is 90% of $100, or $90, the cost of 100 shares is 100 times $90, or $9000, to which add the brokerage, ⅛% of $10000, or $12½, and the sum $9012½, is the entire cost of the stock.

2. What cost 50 shares of N. Y. Central R. R. Stock, at par; brokerage, ¼%?

3. Find the cost of 10 shares of Bank Stock at 104; brokerage ⅛%.

4. What is the cost of $2000 U. S. 6's 5–20, at 112; brokerage ⅛%?

641. 1. A broker has $5010 to invest in bank stock at 25% premium; how many shares can he buy, charging ¼% for brokerage?

ANALYSIS.—Since the stock sells at 25% premium, each share with brokerage will cost $125¼; hence he can buy as many shares as $125¼ are contained times in $5010, or 40 shares.

2. A speculator invested $52000 in Ohio and Mississippi R. R. stock at 25¾, allowing ¼% brokerage; how many shares did he buy?

3. If I invest $2350 in U. S. 6's, '81, at 117⅜, brokerage ⅛%, how many $1000 bonds do I receive?

642. 1. A man bought a number of shares of mining stock at 60, and sold the same at 68, and gained $800 by the transaction. How many shares did he buy?

ANALYSIS.—Since he bought at 60% and sold at 68%, he gained 8% of the par value; hence $800 is 8% of $10000, the par value, and the number of shares at $100 each is 100.

2. Bought R. R. stock at 90, and sold at par, gaining $1000. Required the number of shares.

3. I purchased stock at 110 and sold at 98, losing $1200. How many shares did I buy?

4. A broker bought some stock at par, and sold it at 95, losing $2000. How many shares did he buy?

643. 1. What sum must be invested in California 7's, at 110, to obtain therefrom an annual income of $1400?

ANALYSIS.—Since the annual income is $7 on each share, the number of shares must be equal to $1400 ÷ $7, or 200 shares; and 200 shares at $110 amount to $22000, the required investment.

2. What sum must I invest in stock at 115, paying 10% yearly dividends, to realize an income of $2000?

3. What sum must be invested in N. Y. 7's at 103½, in order to receive therefrom an annual income of $2100?

644. 1. What per cent. of his money will a person obtain by investing in 8% stock at 120?

ANALYSIS.—Since each share costs $120, and pays $8 income, the per cent. will be $\frac{8}{120}$, or $\frac{1}{15}$ of 100%, equal to 6⅔%.

2. What per cent. does stock yield when bought at 90, paying 6% dividends? When bought at 75? At 120?

3. What per cent. of interest does stock yield, which pays 5% semi-annual dividends, if bought at 150? At 140? At 120?

645. 1. What should be paid for stock yielding 6% dividends, in order to realize an annual interest on the investment of 8%?

ANALYSIS.—Since the annual dividend on each share is $6, this must be 8% of the sum required; and if 8% is $6, 1% is $¾, and 100% is $75. Hence the stock must be bought for 75.

2. For what must stock that pays 7% dividends be bought to realize 10% interest? 9%? 8%?

3. For what should Missouri 6's be bought to pay 5% interest? 5½%? 6½%? 8%?

646. 1. How much currency can be bought for $500 in gold, when the latter is at a premium of 10%?

Analysis.—Since $1 in gold is worth $1.10 in currency, $500 in gold is worth 500 times $1.10, or $550. Hence, etc.

2. How much currency can be bought for $200 in gold, when the latter is at a premium of 9%?

3. What is $1000 in gold worth in currency, when the former is at a premium of 12%? Of 9½%? Of 10½%?

647. 1. How much gold can be bought for $440 in currency, when the former is at a premium of 10%?

Analysis.—Since $1 in gold is worth $1.10 in currency, $440 will buy as many dollars in gold as $1.10 is contained times in $440, or $400 in gold. Hence, etc.

2. How much gold selling at 9% premium will $1090 in currency buy? $218? $654?

3. How much gold at 11% premium will $444 buy?

WRITTEN EXERCISES.

648. Find the *cost*

1. Of 220 shares of bank stock, the market value of which is 103¾, brokerage ¼%.

Operation.—(103¾% + ¼%) of $100 = $104, cost of 1 share.
$104 × 220 = $22880, cost of 220 shares. (**640.**)

Formula.—*Entire Cost* = (*Market Value of* 1 *Share* + *Brokerage*) × *No. of Shares.*

2. Find the cost of 350 shares of Western Union Telegraph stock, market value 97¾, brokerage ⅛%.

3. A broker bought for me 15 one-thousand-dollar U. S. 5-20 bonds at 112¼, brokerage ⅛%. What was their cost?

4. My broker sells for me 125 shares of stock at 127%. What should I receive, the brokerage being ¼%?

649. Find the *number of shares*

1. Of bank stock at 105, that can be bought for $25260, including brokerage at ¼%?

OPERATION.—(105% + ¼%) of $100 = $105¼, cost of 1 share.
$25260 ÷ $105¼ = 240, No. of shares. (**641.**)

FORMULA.—*No. of Shares = Investment ÷ Cost of* 1 *Share.*

2. How many shares of N. J. Central R. R. stock at 107¾, brokerage ¼%, can be bought for $27000?

3. How many shares of Mo. 6's at 97¾, brokerage ¼%, will $21560 purchase?

4. Bought Pacific Mail at 29½, and sold at 31¼, paying ⅛% brokerage each way. How many shares will gain $330?

OPERATION.—(31¼% − 29½%) − ¼% = 1½%, gain.
$330 ÷ $1.50 = 220, No. of shares. (**642.**)

FORMULA.—*No. of Shares = Whole Gain or Loss ÷ Gain or Loss per Share.*

5. How many shares of stock bought at 97½ and sold at 102¼, brokerage ⅛% each way, will gain $990?

6. Lost $1680 by selling N. Y. Central at 101 that cost 104. Brokerage being ¼% each way, how many shares did I sell?

7. How many shares of the Bank of Commerce bought *at 110½ and* sold at 116¾, brokerage ⅛% on the purchase *and sale, will* gain $1200?

650. Find the *amount of investment*

1. In U. S. 5's, of '81, at 111, so as to realize therefrom an annual income of $2500?

OPERATION.—$2500 ÷ $5, income on 1 share = 500, No. of shares. $111, price of 1 share × 500 = $55500, investment. (**643.**)

FORMULA.—*Investment = Price of 1 Share × No. of Shares.*

2. What sum must be invested in Tennessee 6's at 85, to yield an annual income of $1800?

3. How much money must be invested in any stock at 105½, which pays 5% semi-annual dividends, to realize an annual income of $2000?

4. What sum invested in stock at $63 per share, will yield an income of $550, the par value of each share being $50, and the stock paying 10% annual dividends?

651. Find the *rate per cent. of income,* realized

1. From bonds paying 8% interest, bought at 110.

OPERATION.—$8, interest per share ÷ $110, cost per share = $.07\frac{3}{11}$, or $7\frac{3}{11}$%. (**644.**)

FORMULA.—*Rate % of Income = Interest per Share ÷ Cost per Share.*

2. If stock paying 10% dividends is at a premium of 12½%, what per cent. of income will be realized on an investment in it?

3. Which will yield the better income, 8% bonds at 110, or 5's at 75?

4. Which is the more profitable, and how much, to buy New York 7's at 105, or 6 per cent. bonds at 84?

5. What per cent. of income does stock paying 10% dividends yield, if bought at 106?

6. What per cent. will stock which pays 5% dividends yield, if bought at a discount of 15%?

7. What rate per cent. of income shall I receive, if I buy U. S. 5's at a premium of 10%, and receive payment at par in 15 years?

652. Find *at what price stock must be bought*

1. That pays 6% dividends, so as to realize an income of 7½% on the investment.

OPERATION.—.06 ÷ .075 = .80 or 80%, price of stock. (**645.**)

FORMULA.—*Price of Stock = Dividend ÷ Rate of Income.*

2. What must be paid for 5% bonds, that the investment may yield 8%?

3. How much premium may be paid on stock that pays 10% dividends, so as to realize 7½% on the investment?

4. What must I pay for Government 5's of '81, that my investment may yield 7%?

5. At what price must stock, of the par value of $50 a share, and that pays 6% dividends, be bought, to yield an income of 7½%?

6. At what price must 6% stock be bought, to pay as good an income as 8% stock bought at par? As 9% stock?

653. Find the *value in currency,*

1. Of $3750 in gold, quoted at 110¼.

OPERATION.—$1.10¼ × 3750 = $4143.75, value in currency. (**646.**)

FORMULA.—*Total Value in Currency = Value of* $1 *in Currency × No. of Dollars in Gold.*

2. Find the value of $4975 in gold, at a premium of 9⅞%?

3. What is the semi-annual interest of $8000 6% gold-bearing bonds worth in currency, when gold is at 111¾?

4. A merchant bought a bill of goods, for which he was to pay $7000 in currency, or $6625 in gold. Gold being at 109⅝, which is the better proposition, and how much in currency?

654. Find the *value in gold*,

1. Of $2150 in currency, when gold is at a premium of 10½%?

OPERATION.—$2150 ÷ 1.105 = $1945.70, value in gold. (**647.**)

FORMULA.—*Total Value in Gold* = *Amt. of Currency* ÷ (1 + *Premium*).

2. What is $4500 in currency worth in gold, when the latter is at a premium of 12½%? At 11½%? At 9½%?

3. How much money must be invested in U. S. 6's at 111, when gold is quoted at 110½, in order to obtain a semi-annual income of $2210 in currency?

4. The Mechanics Bank of New York having $109737.50 to distribute to the stockholders, declares a dividend of 6¼%; what is the amount of its capital?

5. A man owns a house which rents for $1450, and the tax on which is 2¾% on a valuation of $8500. He sells for $15300, and invests in stock at 90, that pays 7% dividends. Is his yearly income increased or diminished, and how much?

6. If I have $36500 to invest, and can buy N. Y. Central 6's at 85, or N. Y. Central 7's at 95, how much more *profitable* will the latter be than the former?

7. Which is the better investment, a mortgage for 3 yr. of $5000, paying 7% interest, and purchased at a discount of 5%, or 50 shares of stock at 95, paying 8% dividends, and sold at the expiration of 3 years at 98?

8. Henry Ivison, through his broker, invested a certain sum of money in New York State 6's at 107½, and twice as much in U. S. 5's, of '81, at 98½, brokerage in each case ½%. The annual income from both investments was $3348. How much did he invest in each kind of stock?

9. A gentleman invested $12480 current funds in U. S. 5-20's of '85, at 104. What will be his annual income in currency when gold is 110?

INSURANCE.

655. ***Insurance*** is a contract of indemnity against loss or damage. It is of two kinds: insurance on *property*, and insurance on *life.*

656. The ***Insurer*** or ***Underwriter*** is the party who takes the risk or makes the contract.

657. The ***Policy*** is the written contract between the parties.

658. The ***Premium*** is the sum paid for insurance, and is a certain per cent. of the sum insured.

659. Insurance business is generally conducted by Companies, which are either *Joint-stock Companies*, or *Mutual Companies*.

A ***Stock Insurance Company*** is one in which the capital is owned by individuals called *stockholders*. They alone share the profits, and are liable for the losses.

A ***Mutual Insurance Company*** is one in which the profits and losses are divided among those who are insured.

Some companies are conducted upon the *Stock* and *Mutual* plans combined and are called *Mixed Companies*.

Insurance on property is of two kinds: *Fire Insurance*, and *Marine and Inland Insurance.*

660. ***Fire Insurance*** is indemnity for loss of property by fire.

661. ***Marine and Inland Insurance*** is indemnity for loss of vessel or cargo, by casualties of navigation on the ocean, or on inland waters.

Transit Insurance refers to risks of transportation by land only, or partly by land and partly by water. The same policy may cover both Marine and Transit Insurance.

Stock Insurance is indemnity for the loss of cattle, horses, etc. Most insurance companies will not take risks to exceed two-thirds or three-fourths the appraised value of the property insured.

When only a part of the property insured is destroyed or damaged, the insurers are required to pay only the estimated loss; and sometimes the claim is adjusted by repairing or replacing the property, instead of paying the amount claimed.

662. The operations are based on the principles of Percentage, the corresponding terms being as follows:

1. The ***Base*** is the *amount* of *insurance.*
2. The ***Rate*** is the *per cent.* of premium.
3. The ***Percentage*** is the *premium.*

ORAL EXERCISES.

663. 1. How much must be paid for insuring a house and furniture for $4000, at $1\frac{1}{4}\%$ premium?

ANALYSIS.—Since the premium is $1\frac{1}{4}\%$, or $\frac{5}{400}$, equal to $\frac{1}{80}$ of the sum insured, the premium on $4000 will be $\frac{1}{80}$ of $4000, or $50. Hence, etc. (**510.**)

2. What will be the annual premium of insurance, at $\frac{3}{4}\%$, on a building valued at $8000?

3. What will be the cost of insuring a quantity of flour, valued at \$1500, at $\frac{4}{5}$%?

4. What must be paid for insuring a case of merchandise, worth \$640, at $2\frac{1}{2}$%?

5. A man owns $\frac{3}{4}$ of a boat-load of corn valued at \$1800, and insures his interest at $1\frac{2}{3}$%. What premium does he pay?

6. Paid \$6 for insuring \$300; what was the rate?

Analysis.—Since the premium on \$300 is \$6, the premium on \$1 is $\frac{1}{300}$ of \$6, or \$.02, equal to 2%. Hence, etc. (**513.**)

7. Paid \$12 for an insurance of \$800; find the rate.

8. Paid \$24 for an insurance of \$1000; find the rate.

9. At 2%, what amount of insurance can be obtained for \$30 premium?

Analysis.—Since 2% is $\frac{2}{100}$ or $\frac{1}{50}$ of the amount insured, \$30, the given premium, is $\frac{1}{50}$ of the amount insured; and \$30 is $\frac{1}{50}$ of 50 times \$30, or \$1500. Hence, etc. (**516.**)

What amount of insurance can be obtained,

10. On a house, for \$75, at 3% premium?

11. On a boat load of flour, for \$150, at $\frac{3}{4}$%?

12. On a car load of horses, for \$90, at $4\frac{1}{2}$%?

13. On a store and its contents, for \$105, at $1\frac{3}{4}$%?

WRITTEN EXERCISES.

664. Find the *Premium*

1. For insuring a building for \$14500, at $1\frac{1}{2}$%.

Operation.—\$14500 × .015 = \$217.50. (**512.**)

Formula.—*Premium = Amount Insured × Rate.*

Find the premium for insuring

2. A house valued at \$5700, at $\frac{3}{4}$%.

3. Merchandise for \$2750, at $\frac{7}{8}$%.

4. A fishing craft, for $15000, at 1¼%.

5. If I take a risk of $25000, at 1¾%, and re-insure ½ of it at 2¼%, what is my balance of the premium?

665. Find the *Rate of Insurance*,

1. If $36 is paid for an insurance of $2400.

OPERATION.—$36 ÷ $2400 = .015, or 1½%. (**515.**)

FORMULA.—*Rate of Insurance = Premium ÷ Sum Insured.*

What is the rate of insurance,

2. If $280 is paid for an insurance of $16000?

3. If $4.30 is paid for an insurance of $860?

4. A tea merchant gets his vessel insured for $20000 in the Royal Company, at ¾%, and for $30000 in the Globe Company, at ½%. What rate of premium does he pay on the whole insurance?

666. To find the *Amount of Insurance*.

1. A speculator paid $262.50 for the insurance of a cargo of corn, at 1½%. For what amount was the corn insured?

OPERATION.—$262.50 ÷ .015 = $17500, the sum insured. (**518.**)

FORMULA.—*Sum Insured = Premium ÷ Rate.*

2. If it cost $93.50 to insure a store for one-half of its value, at 1⅜%, what is the store worth?

3. Paid $245 insurance at 4⅜% on a shipment of pork, to cover ⅝ of its value. What was its total value?

4. A merchant shipped a cargo of flour worth $3597, from New York to Liverpool. For what must he insure it at 3¼%, to cover the value of the flour and premium?

OPERATION.—$3597 ÷ (1 − .03¼) or .9675 = $3717.829. (**520.**)

5. An underwriter agrees to insure some property for enough more than its value to cover the premium, at the rate of 26 cents per $100. If the property is worth $22163, what should be the amount of the policy?

6. For what sum must a policy be issued to insure a dwelling-house, valued at $35000, at $\frac{1}{4}$%, a carriage-house worth $9500, at $\frac{5}{8}$%, and furniture worth $4500, at $\frac{3}{8}$%, 10% being deducted from the premium, which is to be covered by the policy?

7. A person insured his house for $\frac{3}{4}$ of its value at 40 cents per $100, paying a premium of $73.50. What was the value of the house?

8. A dealer shipped a cargo of lumber from Portland to New York, and to cover both the lumber and the premium, took out a policy for $25200, at $1\frac{3}{4}$%. What was the value of the lumber?

9. A merchant had 500 bbl. of flour insured for 80% of their cost, at $3\frac{1}{4}$%, paying $107.25 premium. At what price per barrel must he sell the flour to gain 20%.

LIFE INSURANCE.

667. ***Life Insurance*** is a contract by which a company agrees to pay a certain sum, in case of the death of the insured during the continuance of the policy.

668. A ***Term Life Policy*** is an assurance for one or more years specified.

669. A ***Whole Life Policy*** continues during the life of the insured.

Premiums may be paid annually for life, or in 5, 10, or more installments (called 5-payment, 10-payment policies, etc.), or the entire premium may be paid in one sum in advance.

The premium is computed at a certain sum or rate per $1000 insured, the rate varying with the age of the insured at the time the policy is issued.

A policy of endowment is not in all respects *an insurance policy*, but is rather a covenant to pay a stipulated sum at the end of a certain period to the person named if living.

Most companies issue a form of policy that combines the principles of Term Life Assurance and Simple Endowment, called for brevity Endowment Policy. Hence,

670. An ***Endowment Policy*** is one in which the assurance is payable to the person insured at the end of a certain number of years named, or to his heirs if he die sooner.

An endowment policy is really two policies in one, and the assured pays the premiums of both.

671. A ***Dividend*** is a share of the premiums or profits returned to a policy-holder in a mutual life insurance company.

672. A ***Table of Mortality*** shows how many persons per 1000 at each age are expected to die per annum.

673. A ***Table of Rates*** shows the premium to be charged for $1000 assurance at the different ages.

Such a table is based upon the table of mortality, and the probable rates of interest for money invested, with a margin or loading for expenses.

674. The following condensed table gives data from the American Experience Table of mortality, and the annual premium on the kinds of policies most in use.

AMERICAN EXPERIENCE TABLE—MORTALITY AND PREMIUMS.

AGE.	MORTALITY TABLE.	ANNUAL PREMIUM PER $1000.					
		LIFE TABLE.					ENDOWMENT (AND TERM LIFE). 10 years.
		One Year Term (Net).	*Whole Life.*				
			Payments during life.	Payment for 10 yr. only.	Payment for 5 yr. only.	Single Payment.	
25	8.1	7.75	$19.89	$42.56	$73.87	$326.58	$103.91
26	8.1	7.82	20.40	43.37	75.25	332.58	104.03
27	8.2	7.88	20.93	44.22	76.69	338.83	104.16
28	8.3	7.95	21.48	45.10	78.18	345.31	104.29
29	8.3	8.02	22.07	46.02	79.74	352.05	104.43
30	8.4	8.10	22.70	46.97	81.36	359.05	104.58
31	8.5	8.18	23.35	47.98	83.05	366.33	104.75
32	8.6	8.28	24.05	49.02	84.80	373.89	104.92
33	8.7	8.38	24.78	50.10	86.62	381.73	105.11
34	8.8	8.49	25.56	51.22	88.52	389.88	105.31
35	8.9	8.60	26.38	52.40	90.49	398.34	105.53
40	9.8	9.42	31.30	59.09	101.58	445.55	106.90
45	11.2	10.73	37.97	67.37	115.02	501.69	109.07
50	13.8	13.25	47.18	77.77	131.21	567.13	112.68

The actual net cost of insurance for a single year at each age named in the table, on the mortality assumed, is as many dollars and tenths of a dollar as there are deaths, but discounted for 1 year. Thus, at age 25, deaths 8.1 per 1000, net cost, which is $8.10, discounted at 4½% by the insurance law, $7.75. If this sum, $7.75, is loaded for expenses at, say 25%, the total premium for 1 year is $9.69, if at 40%, then it would be $10,85.

In a Term Life Policy the premium may vary, increasing slightly each year of the term, according to the assumed increasing liability to decease, or it may be *averaged* for the term so as to be the same each year.

The *greater* number of policies now issued by the companies are *on the whole life* plan, with equal annual premiums.

WRITTEN EXERCISES.

675. To find the amount of premium

1. For a life policy of $5000 issued to a person 30 years old.

OPERATION.—$22.70 × 5 = $113.50.

2. For a life policy of $7500, age being 45.

RULE.—*Multiply the premium for* $1000 *assurance by the number of thousands.*

FORMULA.—*Premium = Rate per* $1000 × *No. of thousands.*

3. Find the annual premium for an endowment policy of $10000, payable in 10 years, age 35.

4. What premium must a man aged 30 pay annually for life, for a life policy of $5000?

What premium annually for 10 years?

What premium annually for 5 years?

What premium in a single payment?

OPERATION.

$22.70 × 5000 = $113.50
$46.97 × 5000 = $234.85
$81.36 × 5000 = $406.80
$359.05 × 5000 = $1795.25

ANALYSIS.—Multiply the rate per thousand dollars, found in the Life Table, opposite age 30, by the number of thousands, expressing the hundreds, tens, and units decimally.

5. What annual premium will a man aged 35 years pay to secure an endowment policy for $5000, payable to himself in 10 years, or to his heirs, if death occurs before?

6. If he dies at the beginning of the ninth year, how much will the assurance cost, reckoning simple interest at 6%?

7. How much less would he have paid in the whole life (annual payment) plan, interest included?

8. A man aged 45 insures his life for $7500 on the single-payment plan, and dies 3 yr. 5 mo. afterward. How much less would his insurance have cost him had he insured on the annual payment plan?

9. A person aged 27 takes out a 10-year endowment policy for $5000; the dividends reduce his annual premiums 15% on the average. Computing annual interest at 7% on his premiums, does he gain or lose, and how much?

10. A man aged 35 years took out a life policy for $12000, on the 5-payment plan, and died 3 yr. 6 mo. afterward. What was gained to his estate by insuring, computing compound interest on his payments at 7%, also adding two dividends of $95 each?

TAXES.

676. A ***Tax*** is a sum of money assessed on the person, property, or income of an individual, for any public purpose.

677. A ***Poll Tax*** or ***Capitation Tax*** is a certain sum assessed on every male citizen liable to taxation. Each person so taxed is called a *poll.*

678. A ***Property Tax*** is a tax assessed on property, according to its estimated, or assessed, value.

Property is of two kinds: *Real Property*, or *Real Estate*, and *Personal Property.*

679. ***Real Estate*** is fixed property; such as houses and lands.

680. ***Personal Property*** is of a movable nature; such as furniture, merchandise, ships, cash, notes, mortgages, stock, etc.

681. An ***Assessor*** is an officer appointed to determine the taxable value of property, prepare the assessment rolls, and apportion the taxes.

682. A ***Collector*** is an officer appointed to receive the taxes.

683. An ***Assessment Roll*** is a schedule, or list, containing the names of all the persons liable to taxation in the district or company to be assessed, and the valuation of each person's taxable property.

684. The ***Rate of Property Tax*** is the rate per cent. on the valuation of the property of a city, town, or district, required to raise a specific tax.

WRITTEN EXERCISES.

685. 1. What sum must be assessed to raise $836000 net, after deducting the cost of collection at 5%?

OPERATION.—$836000 ÷ .95 = $880000. (**519.**)

FORMULA.—*Sum to be raised* ÷ (1 — *Rate of Collection*) = *Sum to be Assessed.*

2. What sum must be assessed to raise a net amount of $11123, and pay the cost of collecting at 2%?

3. In a certain district, a school-house is to be built at a cost of $18500. What amount must be assessed to cover this and the collector's fees at 3%?

4. The expense of building a public bridge was $1260.52, which was defrayed by a tax upon the property of the town. The rate of taxation was 3¼ mills on a dollar, and the collector's commission was 3½%. What was the valuation of the property?

5. In a certain town a tax of \$5000 is to be assessed. There are 500 polls, each assessed 75 cents, and the valuation of the taxable property is \$370000. What will be the rate of property tax, and how much will be A's tax, whose property is valued at \$7500, and who pays for 2 polls?

OPERATION.—\$.75 × 500 = \$375, amt. on polls.
\$5000 − \$375 = " " property.
\$4625 ÷ \$370000 = .0125, rate of taxation.
\$7500 × .0125 = \$93.75, A's property tax.
\$93.75 + \$1.50 = \$95.25, A's whole tax.

RULE.—I. *Find the amount of poll tax, if any, and subtract it from the whole amount to be assessed; the remainder is the property tax.*

II. *Divide the property tax by the whole amount of taxable property; the quotient is the rate of taxation.*

III. *Multiply each man's taxable property by the rate of taxation, and to the product add his poll tax, if any; the result is the whole amount of his tax.*

A table such as the following is a great aid in calculating the amount of each person's tax, according to the ascertained rate.

ASSESSOR'S TABLE. (*Rate* .0087.)

Prop.	Tax.	Prop.	Tax.	Prop.	Tax.	Prop.	Tax.
\$1	\$.0087	\$ 9	\$.0783	\$ 80	\$ .696	\$ 700	\$ 6.09
2	.0174	10	.087	90	.783	800	6.96
3	.0261	20	.174	100	.87	900	7.83
4	.0348	30	.261	200	1.74	1000	8.70
5	.0435	40	.348	300	2.61	2000	17.40
6	.0522	50	.435	400	3.48	3000	26.10
7	.0609	60	.522	500	4.35	4000	34.80
8	.0696	70	.609	600	5.22	5000	43.50

6. Find by the table the tax of a person whose property is valued at $3475, the rate being .0087.

OPERATION.—Tax on $3000 = $26.10
" " 400 = 3.48
" " 70 = .609
" " 5 = .0435
" " $3475 = $30.2325, or $30.23.

Find by the table the tax of a person whose property

7. Is $2596, and who pays for 5 polls at $.50.

8. Is $9785, polls 3 at $.75.

9. Is $12356, polls 4 at $1.25.

10. Is $25489, polls 5 at $.95.

11. A tax of $11384, besides cost of collection at 3¼%, is to be raised in a certain town. There are 760 polls assessed at $1.25 each, and the personal property is valued at $124000, and the real estate at $350000. Find the tax rate, make an assessor's table for that rate, and find a person's tax, whose real estate is valued at $6750, personal property at $2500, and who pays for 3 polls.

12. In the above town, how much is B's tax on $15000 real estate, $2750 personal property, and 5 polls?

13. What is C's tax on $9786 and 1 poll?

14. How much tax will a person pay whose property is assessed at $7500, if he pays 1¾% village tax, .001¼% school tax, and ½% State tax?

15. The expense of constructing a bridge was $916.65, which was defrayed by a tax upon the property of the town. The rate of taxation was 2¼ mills on a dollar, and the commission for collecting 3%; what was the assessed valuation of the property of the town?

NOTE.—Amt. to be raised ÷ by rate = valuation.

686. SYNOPSIS FOR REVIEW.

PERCENTAGE—CONTINUED.

- 17. STOCKS.
  - 1. Defs.
    - 1. *Corporation.* 2. *Charter.* 3. *Capital Stock.* 4. *Certificate of Stock, or Scrip.* 5. *Share.* 6. *Stocks.* 7. *Stockholders.* 8. *Par Value.* 9. *Market Value.* 10. *Premium, Discount, Brokerage.* 11. *Stock Broker.* 12. *Stock-jobbing.* 13. *Installment.* 14. *Assessment.* 15. *Dividend.* 16. *Net Earnings.* 17. *Bond.* 18. *Dif. Kinds of U. S. Bonds.* 19. *Coupon.* 20. *Currency.*
  - To find — *Formula.*
    - 2. **648.** *Cost.*
    - 3. **649.** *No. of Shares.*
    - 4. **650.** *Amt. of Investment.*
    - 5. **651.** *Rate % Income.*
    - 6. **652.** *Price to pay Income.*
    - 7. **653.** *Value of Gold in Cur.*
    - 8. **654.** *Value of Cur. in Gold.*
- 18. INSURANCE.
  - 1. Defs.
    - 1. *Insurance.* 2. *Insurer or Underwriter.* 3. *Policy.* 4. *Premium.* 5. *Fire Insurance.* 6. *Marine or Inland Insurance.*
  - 2. Corresponding Terms in Percentage.
  - To find — *Formula.*
    - 3. **664.** *Premium.*
    - 4. **665.** *Rate of Insurance.*
    - 5. **666.** *Amt. of Insurance.*
- 19. LIFE INS.
  - 1. Defs.
    - 1. *Life Insurance.* 2. *Term Life Policy.* 3. *Whole Life Policy.* 4. *Endowment Policy.* 5. *Dividend.* 6. *Table of Mortality.* 7. *Table of Rates.*
  - 2. **675.** Rule. Formula.
- 20. TAXES.
  - 1. Defs.
    - 1. *Tax.* 2. *Poll Tax.* 3. *Property Tax.* 4. *Real Estate.* 5. *Personal Property.* 6. *Assessor.* 7. *Collector.* 8. *Assessment Roll.* 9. *Rate of Property Tax.*
  - To find
    - 2. **685.** Sum to be raised. Formula.
    - 3. **686.** *Amt. of Tax.* Rule, I, II, III.

EXCHANGE

687. ***Exchange*** is the giving or receiving of any sum in one currency for its value in another.

By means of exchange, payments are made to persons at a distance by written orders, called *Bills of Exchange.*

688. ***Exchange*** is of two kinds, *Domestic,* or *Inland*, and *Foreign.*

689. ***Domestic*** or ***Inland Exchange*** relates to remittances made between different places in the same country.

690. ***Foreign Exchange*** relates to remittances made between different countries.

691. A ***Bill of Exchange*** is a written request, or order, upon one person to pay a certain sum to another person, or to his order, at a specified time. An inland bill of exchange is usually called a *Draft.*

692. A ***Set of Exchange*** is a bill drawn in duplicate or triplicate, each copy being valid, until the amount of the bill is paid. These copies are sent by different conveyances, to provide against miscarriage.

693. A ***Sight Draft*** or ***Bill*** is one which requires *payment to be* made "at sight," that is, at the time it is *presented to the* person who is to pay it.

694. A ***Time Draft*** or ***Bill*** is one that requires payment to be made at a certain specified time after date, or after sight.

695. The ***Buyer*** or ***Remitter,*** of a bill is the person who purchases it. The buyer and payee may be the same person.

696. The ***Acceptance*** of a bill or draft is the agreement by the drawee to pay it at maturity. The drawee thus becomes the *acceptor,* and the bill or draft, an *acceptance.*

1. The drawee accepts by writing the word "accepted" across the face of the bill, and signing it.
2. Three days of grace are usually allowed on bills of exchange, as well as on notes. When a bill is protested for non-acceptance, the drawer is bound to pay it immediately.

697. The ***Par of Exchange*** is the estimated value of the coins of one country as compared with those of another. It is either *intrinsic* or *commercial.*

1. The *Intrinsic Par of Exchange* is the comparative value of the coins of different countries, according to their weight and purity.
2. The *Commercial Par of Exchange* is the comparative value of the coins of different countries, according to their market price.

698. The ***Course*** or ***Rate of Exchange*** is the current price paid in one place for bills of exchange on another place.

This price varies according to the relative conditions of trade and commercial credit at the two places between which the exchange is made. Thus, if New York is largely indebted to London, bills of exchange on London will bear a high price in New York.

699. FORMS OF DRAFTS AND BILLS.

A SIGHT DRAFT.

$500. New York, *July* 1, 1874.

At sight, pay to the order of William Thompson, *five hundred dollars, value received, and charge to the acct. of*

Henry J. Carpenter.

To Harris, Jones & Co.,
Cincinnati, O.

Other drafts have the same form as the above, except that instead of the words "at sight," "—— days after sight," or "—— days after date," are used. When the time is *after sight*, it means after acceptance.

SET OF EXCHANGE.

£700. New York, *August* 1, 1874.

At sight of this First of Exchange (Second and Third of the same tenor and date unpaid), pay to the order of Samuel Monmouth, *Seven Hundred Pounds Sterling*, for value received, and charge the same to the account of

Morton, Bliss & Co.

Morton, Rose & Co., London.

The above is the form of the *first bill;* the *second* requires only the change of "First" into "Second," and instead of "Second and Third of the same tenor," etc., "First and Third." The Third Bill varies similarly.

DOMESTIC OR INLAND EXCHANGE.

The course of exchange for inland bills, or drafts, is always expressed by the rate of premium or discount. Time drafts, however, are subject to bank discount, like promissory notes, for the term of credit given. Hence, their cost is affected by both the course of exchange and the rate of discount for the time.

WRITTEN EXERCISES.

700. What is the *cost*

1. Of a sight draft on New Orleans for $1750, at 1¼% premium?

OPERATION.—$\$1750 \times 1.01\frac{1}{4} = \$1771.87\frac{1}{2}$. (**512.**)

$$\text{FORMULA.—}\textit{Cost} = \textit{Face} \times \begin{cases} 1 + \textit{Rate of Premium.} \\ 1 - \textit{Rate of Discount.} \end{cases}$$

2. Of a sight draft on Troy for $1590, at 1½% discount?

3. Of a draft on Boston for $1650, payable in 60 days after sight, exchange being at a premium of 1¾%?

OPERATION.—$1.0175 = Course of Exchange.
$.0105 = Bank Dis. on $1, for 63 da.
$1.007 = Cost of Exchange, for $1.
$1.007 × 1650 = $1661.55, value of Draft.

4. Of a draft on New York at 30 da. for $4720, at 1½% premium?

5. Of a draft on New Orleans, at 90 da., for $5275, int. being 7%, and exchange ½% discount?

6. Find the cost in Philadelphia of a draft on Denver, at 90 da., for $6400, the course of exchange being 101⅜?

7. What must be paid in New York for a draft on San Francisco, at 90 da., for $5600, the course of exchange being 102⅛%?

701. Find the *Face*

1. Of a draft on St. Louis, at 90 da., purchased for $4500, exchange being at 101½%.

OPERATION.—$1.015 = Course of Exchange.
$.0155 = Bank Dis. of $1, for 93 da., at 6%.
$.9995 = Cost of Exchange of $1.
$4500 ÷ .9995 = $4502.25. (**520.**)

2. Of a draft on Richmond at 60 da. sight, purchased for $797.50, interest 7%, premium 2½%.

3. Of a sight draft bought for $711.90, discount 1⅛%.

4. A commission merchant sold 2780 lb. of cotton at 11½ cents a pound. If his commission is 2½%, and the course of exchange 98½%, how large a draft can he buy to remit to his consignor?

5. The Broadway Bank of New York having declared a dividend of 5%, a stockholder in Chicago drew on the bank for the sum due him, and sold the draft at a premium of 1¼%, thus realizing $2283.18¾ from his dividend. How many shares did he own?

6. A man in Rochester purchased a draft on Louisville, Ky., for $5320, drawn at 60 days, paying $5141.78. What was the course of exchange?

7. Received from Savannah 250 bales of cotton, each weighing 520 pounds, and invoiced at 12½ cents a pound. Sold it at an advance of 25%, commission 1½%, and remitted the proceeds by draft. What was the face of the draft, exchange being ½% discount?

FOREIGN EXCHANGE.

702. ***Money of Account*** consists of the denominations or divisions of money of any particular country, in which accounts are kept.

The Act of March 3, 1873, provides that "the value of foreign coin, as expressed in the money of account of the United States, shall be that of the pure metal of such coin of standard value; and the values of the standard coins in circulation, of the various nations of the world, shall be estimated *annually* by the Director of the Mint, and be proclaimed on the first day of January by the Secretary of the Treasury."

703. *The values in United States money of the pure gold or silver representing respectively the monetary units and standard coins of foreign countries are shown in the following Table published by the Secretary of the Treasury,* January 1, 1875.

COUNTRY.	MONETARY UNIT.	STANDARD.	VALUE IN U. S. MONEY.	STANDARD COINS.
Argentine Republic	Peso fuerte	Gold	$1.00	None.
Austria	Florin	Silver	.45,3	Florin.
Belgium	Franc	Gold and silver	.19,3	5, 10, and 20 francs.
Bolivia	Dollar	Gold and silver	.96,5	Escudo, ½ bolivar, and bolivar.
Brazil	Milreis of 1000 reis	Gold	.54,5	None.
British Possessions in N. Am.	Dollar	Gold	1.00	
Bogota	Peso	Gold	.91,2	
Central America	Dollar	Silver	.91,8	Dollar.
Chili	Peso	Gold	.91,2	Condor, doubloon, and escudo.
Cuba	Peso	Gold	.92,5	
Denmark	Crown	Gold	.26,8	10 and 20 crowns.
Ecuador	Dollar	Silver	.91,8	Dollar.
Egypt	Pound of 106 piasters	Gold	4.97,4	5, 10, 25, and 50 piasters.
France	Franc	Gold and silver	.19,3	5, 10, and 20 francs.
Great Britain	Pound sterling	Gold	4.86,6½	½ sovereign and sovereign.
Greece	Drachma	Gold and silver	.19.3	5, 10, 20, 50, and 100 drachmas.
German Empire	Mark	Gold	.23,8	5, 10, and 20 marks.
Hayti	Dollar	Silver	.95,2	
Japan	Yen	Gold	.99,7	1, 2, 5, 10, and 20 yen.
India	Rupee of 16 annas	Silver	.43,6	
Italy	Lira	Gold and silver	.19,3	5, 10, 20, 50, and 100 lire.
Mexico	Dollar	Silver	.99,8	Peso or dollar, 5, 10, 25, and 50 centavo.
Netherlands	Florin	Silver	.38.5	½ florin, florin, and 2½ florins.
Norway	Crown	Gold	.26.8	10 and 20 crowns.
Peru	Dollar	Silver	.91,8	
Porto Rico	Peso	Gold	.92.5	
Portugal	Milreis of 1000 reis	Gold	1.08,4	2, 5, and 10 milreis.
Russia	Rouble of 100 copecks	Silver	.78,4	¼, ½, and 1 rouble.
Sandwich Islands	Dollar	Gold	1.00	
Spain	Peseta of 100 centimes	Gold and silver	.19,3	5, 10, 20, 50, and 100 pesetas.
Sweden	Crown	Gold	.26,8	10 and 20 crowns.
Switzerland	Franc	Gold and silver	.19,3	5, 10, and 20 francs.
Turkey	Piaster	Gold	.04,3	25, 50, 100, 250, and 500 piasters.
Uruguay	Patacon	Gold	.94,9	

704. *Sterling Bills* or ***Sterling Exchange*** are bills on England, Ireland, or Scotland. Such bills are negotiated at a rate fixed without reference to the par of exchange.

Formerly such bills were quoted at a certain rate % above the old par value of a pound sterling, which was $\$4.44\frac{4}{9}$. As this was entirely a fictitious value, and always about 9% below the real value, the course of exchange always appeared to be heavily against this country, and thus tended to impair its credit. By the Act of March, 1873, "all contracts made after the first day of January, 1874, based on an assumed par of exchange with Great Britain of fifty-four pence to the dollar, or $\$4.44\frac{4}{9}$ to the sovereign or pound sterling," are declared *null and void.* The *par of exchange* between Great Britain and the United States is fixed at $4.8665.

705. *Exchanges with Europe* are effected chiefly through the following prominent financial circles: London, Paris, Antwerp, Amsterdam, Hamburg, Frankfort, Bremen, and Berlin.

In exchange on Paris, Antwerp, and Switzerland, the unit is the *franc,* and the quotation shows the number of francs and centimes to the dollar, Federal Money. In exchange on Amsterdam, the unit is the *guilder,* quoted at its value in *cents;* on Hamburg, Frankfort, Bremen, and Berlin, the quotation shows the value of *four reichsmarks* (marks) in *cents.*

WRITTEN EXAMPLES.

706. Find the *cost*

1. Of a bill of exchange on London at 3 days' sight, for £393 15s. 6d, exchange being quoted at 4.89½, and gold at 1.10⅛.

OPERATION.

£393 15s. 6d. = £393.775.

$\$4.895 \times 393.775 = \$1927.529$, *gold* value of bill.

$\$1927.529 \times 1.10\frac{1}{8} = \$2122.69$, value in *currency.*

2. Of a bill of exchange on Liverpool, for £473 5s. 9d par value, in gold.

3. Of a bill of £625 4s. 3d. sterling, at 4.83½, gold 1.09¾.

4. Of a bill on *Paris* for 495 francs, at 5.15 francs to the dollar, in gold.

OPERATION.—495 ÷ 5.15 = $96.12, gold value of the bill.

ANALYSIS.—Since 5.15 francs cost $1, 495 francs will cost as many dollars as 5.15 francs are contained times in 495 francs, or $96.12.

5. Of a bill on *Antwerp* for 697½ francs, at 5.17½ francs to the dollar, in gold.

6. Of a bill on *Geneva*, Switzerland, for 1655 francs, at 5.15⅝, in currency, gold being 1.09⅞.

7. Of a bill on *Frankfort* for 650 marks, at 94⅜, in gold.

OPERATION.—$.94375 ÷ 4 × 650 = $153.36.

ANALYSIS.—Since $.94⅜ is the value of 4 marks, 650 marks are worth 650 times ¼ of $.94⅜, or $153.36.

8. Of a bill on *Berlin* for 1750 marks, quoted at 96¼, in gold.

9. Of a bill of Hamburg for 2155 marks, at 95⅜, in currency, gold being 1.10¼.

10. Of a bill on *Amsterdam* for 2500 guilders, quoted at 41⅝, brokerage ¼%.

OPERATION.—$.41625 × 2500 = $1040.625.
$1040.625 × .00¼ = $2.60, brokerage.
$1040.625 + $2.60 = $1043.225, cost of bill.

11. Of a bill on Amsterdam for 1950 guilders, at 41⅛.

12. Bought exchange on Amsterdam, at 41¼, for 3750 guilders; on Hamburg, at 95½ for 1000 marks; and on London for £500, at $4.85. What was the cost of the whole in currency, gold selling at 109¾?

13. What will it cost to remit directly from Boston to Amsterdam, 12560 guilders, at 41¾?

14. What will be the cost of remitting 13550 marks from New York to Frankfort, exchange selling at 94¼, and gold at 109½; brokerage, ⅛%?

707. What will be the *face*

1. Of a bill of exchange on London that can be bought for $5500, in currency, exchange selling at 4.86, and gold at 1.10?

OPERATION.—$5500 currency ÷ 1.10 = $5000, gold. **(519.)**
$5000 ÷ $4.86 = 1028.806 +.
£1028.806 = £1028 16s. 1½d.

2. Of a bill on *Manchester*, England, that can be bought for $7500, gold; rate of exchange, 4.86?

3. Of a bill on *Berlin* that cost $4000 in gold, exchange 93¾.

OPERATION.—($4000 ÷ $.9375) × 4 = 17066⅔ marks.

ANALYSIS.—Since $.93¾ will buy 4 marks, $4000 will buy 4 times as many marks as $.93¾ is contained times in $4000, or 17066⅔ marks.

4. Of a bill on *Hamburg* that cost $550 in gold, exchange 94⅞?

5. Of a bill on *Frankfort* that cost $395.75 in gold, exchange 95⅛?

6. Of a bill on Geneva, Switzerland, that cost $325 in gold, exchange at 5.17?

OPERATION.—5.17 fr. × 325 = 1680.25 francs.

ANALYSIS.—If $1 will buy 5.17 francs, $325 will buy 325 times 5.17 francs, or 1680.25 francs.

7. A merchant in New Orleans gave $6186, currency, for a bill on Paris, at 5.15½. What was its face?

8. What is the face of a bill on Antwerp, that may be purchased in New York for $2500, exchange at 5.16¾?

ARBITRATION OF EXCHANGE.

708. ***Arbitration of Exchange*** is the process of computing the cost of exchange between two places by means of one or more intermediate exchanges. Such exchange is said to be *indirect*, or *circuitous*.

By this computation the relative cost of direct and indirect exchange is ascertained. Sometimes, owing to the course of exchange between different places, it is more advantageous to remit by the latter than by the former.

Arbitration is either *simple* or *compound*.

709. ***Simple Arbitration*** is that in which there is but one intermediate place.

710. ***Compound Arbitration*** is that in which there are several intermediate places.

WRITTEN EXERCISES.

711. 1. I owe 1500 marks to a merchant in Frankfort. Should I remit directly from New York, or through London, exchange on Frankfort being 94, on London 4.87½, and in the latter place on Frankfort 20.75 marks to the pound, and the London brokerage ⅛%?

OPERATION.—$.94 × 1500 ÷ 4 = $352.50, cost of direct exchange.
1500 marks ÷ 20.75 marks = £72.29.
£72.29 + ⅛% = £72.38.
$4.87½ × 72.38 = $352.85.
$352.85 − $352.50 = $.35, loss by ind. exchange.

2. What will it cost to remit from Boston to Berlin 750 marks, by indirect exchange, through Paris, exchange in New York on Paris being at 5.15, and 4 marks at Paris being worth 4.91 francs, the brokerage being at ¼%?

3. What will it cost to remit 2500 guilders from New York to Amsterdam, through London and Paris, the rates of exchange being as follows: at New York on London 4.83, at London on Paris 24.75 francs to the pound, and at Paris on Amsterdam 2.09 francs to the guilder, brokerage at London and Paris $\frac{1}{8}\%$ each?

OPERATION.

$\$\,x = 2500$ guilders.
1 guilder $= 2.09$ francs.
1 franc (net) $= 1.00\frac{1}{8}$ (with brokerage).
24.75 francs $=$ £1.
£1 (net) $=$ £$1.00\frac{1}{8}$ (with brokerage).
£1 $= \$4.83$.

$$\text{Hence,}\quad \frac{2500 \times 2.09 \times 1.00\frac{1}{8} \times 1.00\frac{1}{8} \times 4.83}{24.75},\quad \text{or}$$

$$\text{By cancellation,}\quad \frac{100 \times 19 \times 1.00\frac{1}{8} \times 1.00\frac{1}{8} \times 1.61}{3} = \$1022.22.$$

ANALYSIS.—Since the members of each equation are equal, the product of the corresponding members of any number of equations are equal; hence, the product of all the second members divided by the product of all the first members except one, must give that member, which is the value required.

4. A merchant in St. Louis directs his agent in New York to draw upon Philadelphia at 1% discount, for \$1500 due from the sale of mdse.; he then draws upon the New York agent, at 2% premium, for the proceeds, after allowing the agent to reserve $\frac{1}{2}\%$ commission. What sum does he realize from his mdse.?

OPERATION.

(x) St. L. $=$ 1500 Philadelphia.
100 Phil. $=$ 99 N. York.
100 N. Y. $=$ 102 St. Louis.
1 $=$ .995 (net proceeds).
By cancellation, $.15 \times 99 \times 102 \times .995 = \$1507.13$.

915

ANALYSIS.—$100 on Philadelphia = $99 on N. Y., and $100 on N. Y. = $102 on St. Louis; and since the agent reserves ½% commission, $1 realized = $.995 net proceeds. Arranging, cancelling, and multiplying, we find the result to be $1507.13.

RULE.—I. *Represent the required sum by* (*x*), *with the proper unit of currency affixed, and place it equal to the given sum on the right.*

II. *Arrange the given rates of exchange so that in any two consecutive equations the same unit of currency shall stand on opposite sides.*

III. *When there is commission for drawing, place* 1 *minus the rate on the left if the cost of exchange is required, and on the right if proceeds are required; and when there is commission for remitting, place* 1 *plus the rate on the right if cost is required, and on the left if proceeds are required.*

IV. *Divide the product of the numbers on the right by the product of the numbers on the left, canceling equal factors, and the result will be the required sum.*

Commission for drawing is commission on the *sale* of a draft; commission for remitting is commission on the *purchase price* of a draft.

The above method of operation is sometimes called the *Chain Rule.*

5. If at New York exchange on London is 4.84½, and at London on Paris it is 25.73 francs to the £, what is the arbitrated course of exchange between New York and Paris?

6. If in London exchange on Paris is 25.71, and in New York on Paris it is 5.15½, what is the arbitrated course of exchange between New York and London?

7. A banker in New York remits $5000 to Liverpool by indirect exchange, through Paris, Hamburg, and Amsterdam, the rates being as follows: in New York on Paris 5.18 fr. to the dollar, in Paris on Hamburg 1.22 fr. to the mark, in Hamburg on Amsterdam 1.70 mark to the guilder, and in Amsterdam 11.83 guilders to the pound sterling. How much sterling will he have in bank at Liverpool, and how much does he gain by indirect exchange, sterling being worth in New York 4.83½?

8. A merchant in Philadelphia owes a correspondent in Paris 35000 francs. Direct exchange on Paris is 5.15; but exchange on London is 4.83, and London exchange on Paris is 25.12. Allowing ½% commission for brokerage at London, which is the most advantageous way to remit, and by how much?

9. An American resident at Amsterdam wishing to obtain funds from the U. S. to the amount of $4500, directs his agent in London to draw on Philadelphia, and remit the proceeds to him in a draft on Amsterdam, exchange on London in Phil. selling at 4.87½, and in London on Amsterdam 11.17½ guilders to the pound sterling. If the agent charges commission at ½% both for drawing and remitting, how much better is this arbitration than to draw directly on the U. S. at 41¼ cents per guilder?

10. A speculator residing in Cincinnati, having purchased 165 shares of railroad stock in New Orleans, at 75%, remits to his agent in N. York a draft purchased at 2% premium, directing the agent to remit the sum due on N. Orleans. Now, if exchange on N. Orleans is at ⅜% discount in N. Y., and the agent's commission for remitting is ⅛%, how much does the stock cost in Cincinnati?

CUSTOM-HOUSE BUSINESS.

712. A ***Custom-House*** is an office established by government for the transaction of business relating to the collection of customs or duties, and the entry and clearance of vessels.

713. A ***Port of Entry*** is a seaport town in which a custom-house is established.

714. The ***Collector of the Port*** is the officer appointed by government to attend to the collection of duties and to other custom-house business.

715. A ***Clearance*** is a certificate given by the Collector of the port, that a vessel has been entered and cleared according to law.

By the *entry* of a vessel is meant the lodgment of its papers in the custom-house, on its arrival at the port.

716. A ***Manifest*** is a detailed statement, or invoice, of a ship's cargo.

No goods, wares, or merchandise can be brought into the United States by any vessel, unless the master has on board a full manifest, showing in detail the several items of the cargo, the place where it was shipped, the names of the consignees, etc.

717. ***Duties*** or ***Customs*** are taxes levied on imported goods.

The general object of such taxes is the support of government, but they are also designed sometimes to protect the manufacturing industry of a country against foreign competition.

718. A ***Tariff*** is a schedule showing the rates of duties fixed by law on all kinds of imported merchandise.

Duties are of two kinds, *Specific* and *Ad Valorem*.

719. A ***Specific Duty*** is a fixed sum imposed on articles according to their weight or measure, but without regard to their value.

720. An ***Ad Valorem Duty*** is an import duty assessed by a percentage of the value of the goods in the country from which they are brought.

Before computing specific duties, certain deductions, or allowances, are made, called *Tare, Leakage, Breakage*, etc.

721. ***Tare*** is an allowance for the weight of the box, cask, bag, etc., that contains the merchandise.

722. ***Leakage*** is an allowance for waste of liquors imported in casks or barrels.

723. ***Breakage*** is an allowance for loss of liquors imported in bottles.

724. ***Gross Weight*** or ***Value*** is the weight or value of the goods before any allowance is made.

725. ***Net Weight*** or ***Value*** is the weight or value of the goods after all allowances have been deducted.

WRITTEN EXERCISES.

726. Find the *Duty*

1. On 355 yds. of carpeting, invoiced at 11s. 6d. per yd., the duty being 50%.

OPERATION.—11s. 6d. = £.575.
£.575 × 355 = £204.125.
$4.8665 (par value of £1) × 204.125 = $993.37.
$993.37 × .50 = $496.68, duty. (**510.**)

2. On 50 hhd. of sugar, each containing 500 lb., at $5\frac{1}{2}$ cts. per lb.; duty $1\frac{3}{4}$ cts. per lb.

3. On 350 boxes of cigars, each containing 100 cigars, invoiced at $7.50 per box; weight, 12 lb. per 1000; duty, $2.50 per lb., and 25% ad valorem.

4. A wine merchant in New York imported from Havre 100 doz. quart bottles of champagne, at \$13 per doz., and 25 casks of sherry wine, each containing 30 gals., at \$2.50 per gal. What is the duty, the rate on the champagne being \$6 per dozen, and on the sherry 60 cents per gal., and 25% ad valorem?

5. Imported from Geneva 25 watches invoiced at \$125 each, and 15 clocks, at \$37.50. What was the duty, the rate being on clocks 25%, and on watches, 35% ad valorem?

6. A liquor dealer receives an invoice of 120 dozen pint bottles of porter, rated at \$.75 per dozen. If 2½% of the bottles are found broken, what will be the duty at 36 cts. per gallon?

7. H. B. Claflin & Co. imported 20 cases of bleached muslins, each case containing 175 pieces of 24 yards each, 1¼ yards wide. What was the duty at 5½ cts. per square yard?

8. What was the duty on 10 cases of shawls, average weight of each case 213½ lb., invoiced at 19375 francs; rate of duty, 50 cts. per lb. and 35% ad valorem? If I pay for the invoice with a bill of exchange bought at 5.15½, and pay charges amounting to \$67.50, what do the shawls cost me in currency, gold selling at 1.10?

9. Olmsted & Taylor, of New York, import from Switzerland 1 case of watches, invoiced at 7125 francs; duty, 25%; charges, fr. 13.50; commissions, 2½%. What was the cost of the watches in U. S. gold?

10. Imported from England 5 cases of cloths and cassimeres, net weight, 695 lb.; value as per invoice, £375 10s. What was the duty in American gold, the rate being 50 cts. per lb. and 35% ad valorem?

EQUATION OF PAYMENTS.

727. ***Equation of Payments*** is the process of finding the average time for the payment of several sums of money due at different times, without loss to debtor or creditor.

728. The ***Equated Time*** is the date at which the several debts may be discharged by one payment.

729. The ***Term of Credit*** is the time at the expiration of which a debt becomes due.

730. The ***Average Term of Credit*** is the time at the end of which the several debts due at different dates, may all be paid at once, without loss to debtor or creditor.

ORAL EXERCISES.

731. 1. The interest of $100 for 3 mo. equals the interest of $50 for how many months?

ANALYSIS.—At the same rate, the interest of $100 equals the interest of $50, or *one-half* of $100, for *twice* the time, or 6 mo.

2. The interest of $20 for 4 mo. equals the interest of $10 for how many mo.? Equals the interest of $5 for how many mo.? Of $1? Of $40? Of $100?

3. The interest of $25 for 6 mo. equals the interest of $5 for how many mo.? Of $10? Of $1?

4. The interest of $10 for 6 mo., and of $100 for 2 mo., taken together, equals the interest of $1 for how many months?

5. If I borrow \$50 for 3 mo., for how many months should I lend \$100 to repay an equal amount of interest?

ANALYSIS.—The interest of \$50 for 3 mo. is the same as the interest of \$1 for 50 times 3 mo., or 150 mo.; and the interest of \$1 for 150 mo. is the same as the interest of \$100 for $\frac{1}{100}$ of 150 mo., or $1\frac{1}{2}$ mo.

6. If I lend \$200 for 3 mo., for how long a time should I have the use of \$50 to balance the favor?

7. If A borrows of B \$1000 for 3 mo., what sum should A lend B for 9 mo. to discharge the obligation?

732. PRINCIPLE.—*The interest and rate remaining the same, the greater the principal the less the time, and the less the principal the greater the time.*

WRITTEN EXERCISES.

733. Find the *average term of credit*

1. Of \$300 due in cash, \$500 due in 3 mo., \$750 due in 8 mo., and \$950 due in 10 mo.

OPERATION

$$\begin{array}{rcl} 300 \times 0 & = & 0 \\ 500 \times 3 & = & 1500 \\ 750 \times 8 & = & 6000 \\ \underline{950} \times 10 & = & \underline{9500} \\ 2500 & &)\underline{17000} \\ & & 6\frac{4}{5}\text{ mo.} \end{array}$$

ANALYSIS.—On \$300, the first payment, there is no interest, since it is due in cash; the int. of \$500 for 3 mo., is the same as the int. of \$1 for 1500 mo.; the int. of \$750 for 8 mo. is the same as that of \$1 for 6000 mo.; and the int. of \$950 for 10 mo. is the same as the int. of \$1 for 9500 mo. Therefore, the whole amt. of int. is that of \$1 for 1500 mo. + 6000 mo. + 9500 mo., or 17000 mo.; but the whole debt is \$2500; and the int. of \$1 for 17000 mo. is equal to the int. of \$2500 for $\frac{1}{2500}$ of 17000 mo., or $6\frac{4}{5}$ mo.

2. Find the average term of credit of \$800 due in 1 mo., \$750 due in 4 mo., and \$1000 due in 6 mo.

RULE.—I. *Multiply each payment by its term of credit, and divide the sum of the products by the sum of the payments; the quotient is the average term of credit.*

II. (To find the equated time of payment,) *Add the average term of credit to the date at which the several credits begin.*

3. On the first day of December, 1876, a man gave 3 notes, the first for $500 payable in 3 mo.; the second for $750 payable in 6 mo.; and the third for $1200 payable in 9 mo. What was the average term of credit, and the equated time of payment?

4. Bought merchandise Jan. 1, 1875, as follows: $350 on 2 mo., $500 on 3 mo., $700 on 6 mo. What is the equated time of payment?

5. A person owes a debt cf $1680 due in 8 months, of which he pays ⅓ in 3 mo., ¼ in 5 mo., ⅕ in 6 mo., and ⅙ in 7 mo. When is the remainder due?

6. Bought a bill of goods, amounting to $1500 on 6 months' credit. At the end of 2 mo., I paid $300 on account, and 2 mo. afterward, paid $400 on account, at the same time giving my note for the balance. For what time was the note drawn?

OPERATION.

$$\begin{array}{lr} 300 \times 4 = & 1200 \\ \underline{400} \times 2 = & \underline{\ \ 800} \\ 800 &)\underline{2000} \\ & 2\frac{1}{2} \end{array}$$

$(6 \text{ mo.} - 4 \text{ mo.}) + 2\frac{1}{2} \text{ mo.} = 4\frac{1}{2} \text{ mo.}$

ANALYSIS.—$300 paid 4 mo. before it is due, and $400, 2 mo. before it is due, are equivalent to the use of $1 for 2000 months, or the use of $800 (the balance) for 2½ mo. beyond the original time. Hence, the note was drawn for 4½ mo. after the second payment.

7. On a debt of $2500 due in 8 mo. from Feb. 1, the following payments were made: May 1, $250, July 1, $300, and Sept. 1, $500. When is the balance due?

8. Find the average term of credit, and the equated time of payment from Dec. 15, of $225 due in 35 da., $350 due in 60 da., and $750 due in 90 da.

9. Dec. 1, 1874, purchased goods to the amount of $1200, on the following terms: 25% payable in cash, 30% in 3 mo., 20% in 4 mo., and the balance in 6 mo. Find the equated time of payment, and the cash value of the goods, computing discount at 7%.

734. To find the equated time when the terms of credit begin at different dates.

1. J. Prince bought goods of W. Sloan as follows: June 1, 1874, amounting to $350 on 2 mo. credit; July 15, 1874, $400, on 3 mo. credit; Aug. 10, $450, on 4 mo. credit; Sept. 12, $600, on 6 mo. credit. What is the equated time of payment?

OPERATION.

$350	due	Aug. 1,	350 × 0	=	0
400	"	Oct. 15,	400 × 75	=	30000
450	"	Dec. 10,	450 × 131	=	58950
600	"	Mar. 12,	600 × 223	=	133800
			1800		1800)222750
					123¾

Hence the equated time is 124 da. from Aug 1, or *Dec. 3.*

ANALYSIS.—Computing the terms of credit from Aug. 1, the *earliest date* at which any of the debts become due, we find the terms of credit to be from Aug. 1 to Oct. 15, 75 da.; to Dec. 10, 131 da., and to March 12, 223 da. The average term of credit is *therefore 124 da. from* Aug. 1, and the equated time Dec. 3.

PROOF.—Assume as the standard time the latest date, March 12. The operation will then be as follows:

$$\begin{array}{rcrcr}
350 & \times & 223 & = & 78050 \\
400 & \times & 148 & = & 59200 \\
450 & \times & 92 & = & 41400 \\
600 & \times & 0 & = & 0 \\
\hline
 & & & 1800) & 178650 \\
\hline
 & & & & 99\frac{1}{4}
\end{array}$$

Hence, the equated time is 99 da. previous to March 12, or *Dec. 3.*

2. Peake & Co. sell to Wm. Jones the following bills of goods: March 1, 1875, on 60 da., $800; April 15, on 30 da., $350; May 20, on 4 mo., $3800.

What is the equated time for settlement?

RULE.—I. *Find the date at which each debt becomes due.*

II. *From the earliest of these dates as a standard compute the time to each of the others.*

III. *Then find the average term of credit and equated time as in* (**733**).

PROOF.—*Compute the terms of credit backward from the latest date, and subtract the average time from that date for the equated time.*

If the earliest date is not the *first* of the month, it is more convenient to assume the *first* of the month as the standard date.

3. Bought mdse. as follows: Jan. 15, 1876, on 4 mo., $375; Feb. 3, on 60 da., $550; March 25, on 4 mo., $1100; April 2, on 30 da., $250. Find the equated time.

4. Ira Blunt, of Gadsden, Ala., bought of Opdyke & Co. the following bills of goods on 4 months' credit:

Jan. 1, 1874, $650; Feb. 10, $380; March 12, $900; March 18, $350; April 3, $600.

April 5, he discounted his bills at 2% per month. Find the *equated time of* payment, and the discount.

5. JAMES SMITH *to* THOMAS BROWN, *Dr.*

March 10,	1874.	To mdse.	$835.
" 18,	"	" "	320.
" 26,	"	" "	475.
April 5,	"	" "	600.
" 12,	"	" "	250.

Allowing 30 days' credit on each of the bills, what is the equated time of payment?

6. Purchased goods as follows:

Sept. 15,	1875,	a bill of	$275,	on 3 mos.
Oct. 10,	"	"	351.50,	" 60 da.
" 28,	"	"	415.75,	" 30 da.
Nov. 3,	"	"	500,	" 4 mos.
Dec. 15,	"	"	710,	" 3 mos.

What was due on this account Aug. 10, 1876, computing interest at 7%?

7. I have four notes, as follows: the first for $425, due April 1, 1875; the second for $615, due May 10, 1875; the third for $1500, due May 28, 1875; and the fourth for $750, due June 10, 1875.

At what date should a single note be made payable, to be given in exchange for the four notes?

AVERAGING ACCOUNTS.

735. An ***Account*** is a written statement of debit and credit transactions, with their respective dates.

Debit means what is owed by the person with whom the account is kept; *credit*, what is due to him from the person keeping the account.

736. To ***Average an Account*** is to find, either the *equated time* of paying the balance, or the *cash balance* at any given time.

Each item of a book account should draw interest from the time it becomes due.

WRITTEN EXERCISES.

737. 1. Find the *equated time* of paying the balance of the following account.

Dr. WILLIAM SAMPSON. *Cr.*

1875.			1875.		
Jan. 11	To mdse. . . .	$750	Feb. 10	By draft at 60 da.	$500
Feb. 1	" " at 3 mo.	600	Mar. 3	" cash . . .	700
Mar. 15	" " at 6 mo.	1500	Apr. 15	" " . . .	300
May 3	" " at 4 mo.	900			

OPERATION I. (*Method by Products.*)

Due.	*Amt.*	*Days.*	*Product.*	*Paid.*	*Amt.*		
Jan. 11.	750 ×	10 =	7500	Apr. 14.	500 ×	103 =	51500
May 1.	600 ×	120 =	7200	Mar. 3.	700 ×	61 =	42700
Sept. 15.	1500 ×	257 =	385500	Apr. 15.	300 ×	104 =	31200
" 3.	900 ×	245 =	220500		1500		125400
	3750		685500				
	1500		125400				
	2250		)560100				

$248\frac{42}{45}$, or 249 da.

Balance due 249 da. from Jan. 1, or *Sept. 7.*

ANALYSIS.—Assuming for convenience Jan. 1 as the standard date, we find as in **734** the term of credit of each debit amount; and, reckoning from the same date, the time to each credit amount. Multiplying each amount by its time in days, and adding the debit and credit products, we find the number of days' interest of $1 due to the debtor, and the number of days' interest of $1 he has already received. The difference, 560100, shows the number of days' interest of $1 still due, and as the balance is $2250, the time must be $\frac{1}{2250}$ of 560100 da., or 249 da. Hence, the *equated time* is 249 da. from Jan. 1, or *Sept. 7.*

OPERATION II. (*Method by Interest.*)

Dr.

$750	to Jan. 11 (from Jan. 1)	= 10 da.,	int. at 1% per mo.	$2.50
600	" Feb. 1 + 3 mo.	= 4 mo.	" "	24.00
1500	" Mar. 15 + 6 mo.	= 8 mo. 14 da.,	" "	127.00
900	" May 3 + 4 mo.	= 8 mo. 2 da.,	" "	72.60
$3750				$226.10

Cr.

$500	to Feb. 10 + 63 da.	= 3 mo. 13 da.,	int. at 1% per mo.	$17.17
700	" Mar. 3	= 2 mo. 2 da.,	" "	14.47
300	" Apr. 15	= 3 mo. 14 da.,	" "	10.40
$1500				$42.04

$226.10 − $42.04 = $184.06, int. at 1% per mo. due.
Int. of balance, $2250, for 1 mo., at 1% = $22.50.
Hence, $184.06 ÷ $22.50 = 8.18+ mo., or 8 mo. 6 da.
8 mo. 6 da. from Jan. 1, or Sept. 7, *Equated Time.*

In this operation, 12% per annum or 1% per mo. is assumed for convenience; since the int. at 1% per mo. is as many hundredths as there are months, and one-third as many thousandths as there are days. Thus, the int. of $249 for 2 mo. 9 da. is $4.98 + $.747 = $5.727 (**571**).

2. Find the *equated time* of the following:

Dr. WILLIAM SIMPSON. *Cr.*

1874.			1874.		
Aug. 5	To mdse. at 3 mo.	$720	Oct. 10	By cash . . .	$500
Sept. 10	" " " 2 "	850	Dec. 15	" draft at 60 da.	450
Nov. 3	" "	1200	" 25	" cash . . .	900
1875.			1875.		
Jan. 20	" sundr's at 5 mo.	620	Jan. 3	" " . . .	250

RULE 1.—I. *Find the date at which each debit item is due, and each credit item is paid or due.*

II. *Take the first day of the month in the earliest date on either side of the account* as a standard date, and *mul-*

tiply each sum due or paid by the number of days between its time and the standard date.

III. *Add the products, and their difference divided by the balance due will give the number of days between the standard date and the equated time.* Or,

RULE 2.—*Find the time of each item from the standard date as before, and compute the interest on each at 1% a month. The difference between the amount of interest on each side divided by the interest of the balance at 1% for one month will be the equated time.*

When the terms of credit are long, Rule 2. gives the shorter method.

3. Find the equated time of the following, allowing 60 da. credit on each debit item:

Dr. John Driscoll. *Cr.*

1877.			1877.		
June 1	To mdse. . .	$950	Aug. 1	By cash . .	$700
July 6	" " . .	300	Sept.20	" " . .	1000
Sept. 8	" " . .	1900	Nov. 1	" " . .	1200
Oct. 20	" " . .	2600			

4. What is the equated time for the payment of the balance of the following account, allowing 4 months' credit on all the debit items?

Dr. Dodd, Brown & Co. *Cr.*

1878.			1878.		
Jan. 20	To mdse. . .	$570	Feb. 14	By mdse. . .	$490
" 28	" " . .	300	Mar. 1	" cash . .	1000
Feb. 11	" " . .	720	Apr. 2	" " . .	1800
" 26	" " . .	835			
Mar. 10	" " . .	1150			
" 28	" " . .	960			
Apr. 15	" " . .	475			

738. 1. Find the *cash balance* of the following account on the 22d of August, allowing interest at 6%:

Dr. GEORGE HAMMOND. *Cr.*

1875.			1875.		
Mar. 15	To mdse., at 3 mo.	$600	May 10	By cash . .	$300
Apr. 3	" " " 4 mo.	700	July 1	" " . .	400
May 10	" " " 6 mo.	1000	Aug. 15	" " . .	500

OPERATION.—By averaging the account, the equated time for paying the balance, $1100, is found to be *Nov.* 4. (**734.**)

True present worth of $1100 for 74 da. (from Aug. 22 to Nov. 4) is $1086.60, or cash balance Aug. 22.

Or, by *Interest Method,* as follows:

Dr.

Int. of $600, from June 15 to Aug. 22, 68 da., $6.71 (**574.**)
" " 700, " Aug. 3 " " 19 da., 2.19
$8.90

Cr.

Int. of $1000, from Aug. 22 to Nov. 10, 80 da., $13.15
" " 300, " May 10 " Aug. 22, 104 da., 5.13
" " 400, " July 1 " " 52 da., 3.42
" " 500, " Aug. 15 " " 7 da., 58
$22.28
8.90

Balance of interest due Hammond, $13.38
$1100 − $13.38 = $1086.62, Cash Balance, Aug. 22.

ANALYSIS.—Charge Hammond with interest on each debit item from the time it is due to date of settlement, and credit him with interest on each sum paid from the date of payment to date of settlement, also on each debit item which becomes due after the date of settlement. Hence, he is entitled to interest on $1000 from Aug. 22 to Nov. 10. As the balance of interest is in favor of Hammond, it must be deducted from the balance of the account, to obtain the cash balance. There is a slight difference in the results, but the interest method is the more accurate. By the use of *Interest Tables,* it is also the shorter of the two methods.

RULE 1.—I. *Average the account, and find the equated time of payment of the balance.*

II. *If the date of settlement is prior to the equated time, find the present worth of the balance of account for the cash balance; if later, find the interest of the balance of account for the intervening time, and add it to find the cash balance.* Or,

RULE 2.—*Find the interest on each debit and credit item, from the time it is due or paid to the date of settlement, placing on the same side of the account the interest on each item due prior to the date of settlement, and on the opposite side the interest on each item due after the date of settlement. If the balance of interest is on the same side as the balance of the account, add it, if on the other side subtract it; and the result will be the cash balance at the date of settlement.*

2. I owe $1500 due May 1, and $750 due Aug. 15. If I give my note at 30 da. for $450, June 1, and pay $370 in cash July 15, what is the equated time for paying the balance; and what would be due in cash Dec. 10, allowing interest at 7%?

3. When is the balance of the following account due per average?

Dr. O. B. TIMPSON. *Cr.*

1875.			1875.		
Aug. 10	To mdse. @ 60 da. .	$751.35	Oct. 3	By cash	$300.00
Sept. 5	" " @ 30 da. .	425.00	Nov.15	" note @ 90 da. .	450.00
Nov. 1	" " @ 90 da. .	927.83	Dec. 20	" cash	500.00
Dec. 5	" " @ 30 da. .	1200.00			

4. What is the cash balance of the above account Jan. 1, 1876, allowing interest at 10%?

5. Find the equated time, and cash balance July 1, of the following, allowing 7% interest:

Dr. THOMAS SMITH. *Cr.*

Jan. 4	To mdse. @ 4 mo.	$1600	Feb. 1	By mdse. @ 4 mo.	$500
" 6	" " @ 3 mo.	1500	Mar. 2	" cash . . .	2000
Apr. 10	" " @ 60 da.	3000	" 25	" " . . .	3150
" 28	" " @ 30 da.	2500	Apr. 16	" " . . .	800

6. Average the following account, and find for what amount a note at 60 days should be given Aug. 1, to pay the balance, interest at 6%.

Dr. ORSON HINMAN. *Cr.*

1875.			1875.		
Apr. 2	To charges .	$87.25	Feb. 25	By mdse. @ 8 mo.	$600
May 15	" " .	35.75	Mar. 3	" " @ 6 "	300
			Apr. 1	" " @ 6 "	500

739. 1. Average the following *Account Sales*, and find when the net proceeds are due. **(543.)**

Account Sales of 1200 bbls. of flour received from Smith, Tyler & Co., Cincinnati.

Date.	Buyer.	Quantity.	Price.	Amount.
1876.				
May 1	J. Brooke	300 bbl.	@ $5.50, 3 mo.	$1650.00
June 5	W. Long	450 "	@ 6.20, 4 mo.	2790.00
" 15	A. Bruce	250 "	@ 6.50, 6 mo.	1625.00
July 1	W. Case	200 "	@ 5.75, 2 mo.	1150.00
				$7215.00

CHARGES.

Apr. 28.	Freight	$674.50	
" "	Cartage	37.50	
May 1.	Storage	191.00	
	Commission on $7215, @ 2¼% . . .	162.34	
	Total charges		$1065.34
	Net proceeds due per average		$6149.66

OPERATION.

Average of sales, found by the method of Equation of Payments, *Oct. 1*, which is the date at which the commission is due.

Average of charges, including commission (Oct. 1), *May 22*. Equated time of $7215 due Oct. 1, and $1065.34 due May 22, *Oct. 21*, date when the net proceeds are due.

RULE.—I. *Average the sales alone, and the result will be the date to be given to the commission and guaranty.*

II. *Make the sales the credits and the charges the debits, and find the equated time for paying the balance.*

2.. Make an account sales, and find the net proceeds and the time the balance is due:

Wm. Brown, of N. York, sold on acct. of J. Berry, of Chicago, June 1, 350 bu. Winter Wheat, @ $1.35, at 60 da.; June 15, 275 bu. Spring Wheat, @ $1.75, at 90 da.; July 3, 1260 bu. Indian Corn, @ $.79, at 6 mo.; and July 10, 375 bu. Rye, @ $1.02, at 3 mo. Paid freight, May 28, $567.50; cartage, May 30, $22.50; insurance, June 5, $56.25; and charged com. at 3¼%, and 1¾% for guaranty.

3. Sold on account of Brown, Sampson & Co., at 6 mo.: Oct. 1, 1874, 13 hhd. sugar, averaging 1520 lb., @ $.12½; Oct. 5, 15 chests Hyson Tea, each 95 lb., @ $1.05. Paid charges: Oct. 3, Insurance, $85; Oct. 10, Cooperage, etc., $24.50; Oct. 20, Cartage, $125. Charged commission and guaranty, 4½%. Make an account sales, and find the equated time for paying the net proceeds.

740. SYNOPSIS FOR REVIEW.

PERCENTAGE—CONTINUED.

- 21. EXCHANGE.
  - 1. Defs.
    - 1. *Exchange.* 2. *Domestic Exchange.* 3. *Foreign Exchange.* 4. *Bill of Exchange.* 5. *Set of Exchange.* 6. *Sight Draft or Bill.* 7. *Time Draft or Bill.* 8. *Buyer or Remitter.* 9. *Acceptance.* 10. *Par of Exchange.* 11. *Course or Rate of Exchange.*
  - 2. Forms. 1. *A Sight Draft.* 2. *Set of Exchange.*
  - 3. Inland Exch.
    - 1. **700.** To find *Cost of Draft. Formula.*
    - 2. **701.** To find *Face of Draft.*
  - 4. Foreign Exch'ge.
    - 1. Defs.
      - 1. *Money of Account.*
      - 2. *Sterling Bills, or Exchange.*
    - 2. Exchange with Europe—how effected.
    - 3. **706.** To find *Cost of Bill.*
    - 4. **707.** To find *Face of Bill.*
  - 5. Arbitration of Exch'ge.
    - 1. Defs.
      - 1. *Arbitration of Exchange.*
      - 2. *Simple Arbitration.*
      - 3. *Compound Arbitration.*
    - 2. Rule, I, II, III, IV.
- 22. CUSTOM-HOUSE BUSINESS.
  - 1. Defs.
    - 1. *Custom House.* 2. *Port of Entry.* 3. *Collector.* 4. *Clearance.* 5. *Manifest.* 6. *Duties or Customs.* 7. *Tariff.* 8. *Specific Duty.* 9. *Ad Valorem Duty.* 10. *Gross W'ght.* 11. *Net Weight-*
  - 2. **726.** To find the Duty.
- 23. EQUATION OF PAYM'TS.
  - 1. Defs.
    - 1. *Equation of Payments.* 2. *Equated Time.* 3. *Term of Credit.* 4. *Average Term of Credit.*
  - 2. Principle.
  - 3. **733.** Rule, I, II.
  - 4. **734.** Rule, I, II, III. Proof.
- 24. AVERAGING ACCOUNTS.
  - 1. Defs. 1. *Account.* 2. *To Average an Acct.*
  - 2. **737.** Rule 1, I, II, III. Rule 2.
  - 3. **738.** Rule 1, I, II. Rule 2.
  - 4. **739.** Rule, I, II.

RATIO

ORAL EXERCISES.

741. 1. A father is 30 years old, and his son 6; how many times as old as the son is the father?

2. 30 are how many times 6? $30 \div 6 = ?$

3. What part of $30 are $6? Of 20 cents are 5 cents?

4. What is the relation of 8 to 2? Of 40 rd. to 4 rd.?

5. What relation has 12 to 3? 60 lb. to 20 lb.?

Compare the following, and give their relative values.

6. 75 with 5.
7. 25 with $6\frac{1}{4}$.
8. 1 with 7.
9. $\frac{1}{7}$ with 7.
10. $2\frac{1}{2}$ with $3\frac{1}{2}$.
11. .9 with .3.
12. $.6 with $.2.
13. .42 with .3.
14. $\frac{2}{3}$ with $\frac{3}{4}$.

DEFINITIONS.

742. ***Ratio*** is the relation between two numbers of the same unit value, expressed by the quotient of the first divided by the second. Thus the ratio of 12 to 4 is $12 \div 4 = 3$.

743. The ***Sign*** of ratio is the colon (:), or the sign of division with the line omitted.

Thus, the ratio of 9 to 3 is expressed $9 : 3$, or $9 \div 3$, or in the form of a fraction $\frac{9}{3}$, and is read, the ratio of 9 to 3, or 9 divided by 3.

744. The ***Terms*** of a ratio are the two numbers compared.

745. The ***Antecedent*** is the first term, or dividend.

746. The ***Consequent*** is the second term, or divisor.

747. The ***Value*** of a ratio is the *quotient* of the antecedent divided by the consequent, and is an *abstract* number.

Thus, in the ratio \$18 : \$6, \$18 and \$6 are the *terms* of the ratio; \$18 is the *antecedent*; \$6 is the *consequent*; and 3, the quotient of \$18 ÷ \$6, is the *value* of the ratio.

748. A ***Simple Ratio*** is the ratio of two numbers; as 10 : 5.

749. A ***Compound Ratio*** is the ratio of the products of the corresponding terms of two or more simple ratios.

Thus the ratio compounded of the simple ratios,

$$\left.\begin{matrix} 8 : 4 \\ 9 : 12 \end{matrix}\right\} \text{ may be expressed } \left\{\begin{matrix} (8:4)\times(9:12) \\ \text{or, } 8\times9 : 4\times12 \end{matrix}\right\} = 72 : 48;$$

$$\text{Or,} \qquad \tfrac{8}{4} \times \tfrac{9}{12} = \tfrac{3}{2} = 3 : 2.$$

When the multiplication is performed the *result* is a *simple* ratio.

750. The ***Reciprocal*** of a ratio is 1 divided by the ratio (**196**), or it is the consequent divided by the antecedent. Thus the ratio of 8 to 9 is 8 : 9, or $\frac{8}{9}$, and its *reciprocal* is $\frac{9}{8}$.

The ratio of two *fractions* is obtained by reducing them to a common denominator, when they are to each other as their numerators (**241**).

If the terms of a ratio are denominate numbers, they must be reduced to the same unit value.

751. From the preceding definitions and illustrations are deduced the following

FORMULAS.—1. *The Ratio = Antecedent ÷ Consequent.*
2. *The Consequent = Antecedent ÷ Ratio.*
3. *The Antecedent =* Consequent × *Ratio.*

752. Since the antecedent is a dividend, and the consequent a divisor, any change in either or both of the terms of a ratio will affect its value according to the laws of division or of fractions (**200**), which laws become the

GENERAL PRINCIPLES OF RATIO.

1. *Multiplying the antecedent*, or *Dividing the consequent*,	*Multiplies the ratio.*
2. *Dividing the antecedent*, or *Multiplying the consequent*,	*Divides the ratio.*
3. *Multiplying or dividing both antecedent and consequent by the same number*,	*Does not change the ratio.*

753. These principles may be embraced in one

GENERAL LAW.

A change in the antecedent produces a LIKE *change in the ratio; but a change in the consequent produces an* OPPOSITE *change in the ratio.*

EXERCISES.

754. 1. Express the ratio of 11 to 4; of 16 to 2; of 20 to $6\frac{2}{3}$; of \$36 to \$12; of 9 lb. to 27 lb.; of $4\frac{1}{2}$ bu. to 9 bu.

2. Can you express the ratio between \$15 and 5 lb.? Why not?

3. Indicate the ratio of 18 to 20 in *two forms.* What are the *terms* of the ratio? The *antecedent?* The *consequent?* The *kind* of ratio? The *value* of the ratio.

In like manner express, analyze, and give the value,

4. Of 80 to 120; of $12\frac{1}{2}$ to $37\frac{1}{2}$; of $16\frac{1}{8}$ to $\frac{3}{8}$.

5. *Of 5.2 to 1.3; of* $\frac{3}{4}$ to $\frac{9}{10}$; of $\frac{2 \times 27 \times 42}{12 \times 4 \times 126}$.

6. The antecedents of a ratio are 7 and 10, and the consequents, 5 and 4. What is the value of the ratio?

7. The first terms of a ratio are 18, 12, and 30, the second, 54, 6, and 15. What is the kind of ratio? Express in *three forms*. Find its value in the lowest terms.

Solve, and state the *formula* applied to the following:

8. The consequent is $3\frac{1}{4}$, the antecedent $\frac{13}{28}$; what is the ratio?

9. The antecedent is 60, the ratio 7; what is the consequent?

10. The consequent is 6.12\frac{1}{2}$, the ratio $\frac{13}{28}$; what is the antecedent?

11. The ratio is $2\frac{2}{5}$, the antecedent $\frac{1}{2}$ of $\frac{5}{8}$; what is the consequent?

12. The ratio is 6, the consequent 1 wk. 3 da. 12 hr.; what is the antecedent?

13. Express the ratio of 120 to 80, and give its value in the lowest terms.

14. Make such changes in the last example as will illustrate PRIN. 1.

15. With the same example, illustrate PRIN. 2.

16. Illustrate by the same example PRIN. 3.

17. Find the *reciprocal* of the ratio of 75 to 15.

18. Find the *reciprocal* of the ratio of 2 qt. 1 pt. to 4 gal. 1 qt. 1 pt.

What is the ratio

19. Of 40 bu. 4.5 pk. to 25 bu. 2 pk. 1 qt.

20. Of 6 A. 110 P. to 10 A. 60 P.

21. Of 25 lb. 11 oz. 4 pwt. to 19 lb. 5 oz. 8 pwt.

22. *Of* $\frac{12\frac{1}{2}}{\frac{4}{7}}$ to $\frac{\frac{2}{3}\text{ of }\frac{3}{4}}{\frac{1}{2}}$.

PROPORTION

ORAL EXERCISES.

755. 1. What is the ratio of 4 to 2? Of 6 to 1? Of 14 to 7? Of 21 to 3?

2. Find two numbers that have the same quotient as $8 \div 2$. As $27 \div 3$. As $16 \div 4$. As $30 \div 6$. As $4 \div \frac{1}{2}$.

3. Express in the form of a fraction the ratio of 26 to 13. Of 32 to 8.

4. Express in both forms the ratio of two other numbers equal to the ratio of 10 to 2. Of 15 to 5. Of 12 to 3.

5. If 4 stamps cost 12 cents, what will 20 stamps cost at the same rate?

6. What number divided by 12, gives the same quotient as $20 \div 4$?

7. What number has the same ratio to 12, that 20 has to 4?

8. To what number has 48 the same ratio that 80 has to 5? That 24 has to 3?

9. The ratio of 20 to 5 is the same as the ratio of what number to 4? To 6? To $5\frac{1}{2}$? To $6\frac{1}{4}$?

10. The ratio of 45 to 9 is the same as the ratio of 15 to what number? Of 30 to what number?

11. 28 is to 7 as 8 is to what number?

12. 56 is to 8 as what number is to 5?

13. $63 \div$ what number equals the ratio of 36 to 4?

DEFINITIONS.

756. A ***Proportion*** is an equation in which each member is a ratio; or it is an equality of ratios.

757. The equality of the two ratios may be indicated by the sign = or by the double colon ::

Thus, we may indicate that the ratio of 8 to 4 is equal to that of 6 to 3, in any of the following ways:

$8 : 4 = 6 : 3,\quad 8 : 4 :: 6 : 3,\quad \frac{8}{4} = \frac{6}{3},\quad 8 \div 4 = 6 \div 3.$

This proportion, in any of its forms, is read, *The ratio of 8 to 4 is equal to the ratio of 6 to 3, or, 8 is to 4 as 6 is to 3.*

758. Since each ratio consists of two terms, every proportion must consist of at least *four terms.* Each ratio is called a *Couplet,* and each term is called a *Proportional.*

759. The ***Antecedents*** of a proportion are the first and third terms, that is, the antecedents of its ratios.

760. The ***Consequents*** are the second and fourth terms, or the consequents of its ratios.

761. The ***Extremes*** are the first and fourth terms.

762. The ***Means*** are the second and third terms.

In the proportion $8 : 4 :: 10 : 5$, 8, 4, 10, and 5 are the *proportionals;* 8 : 4 is the first *couplet,* 10 : 5 the second couplet; 8 and 10 are the *antecedents,* 4 and 5 are the *consequents;* 8 and 5 are the *extremes,* 4 and 10 are the *means.*

Three numbers are proportional, when the ratio of the first to the second is equal to the ratio of the second to the third. Thus the numbers 4, 10, and 25 are proportional, since $4 : 10 = 10 : 25$, the ratio of each couplet being $\frac{2}{5}$.

When three numbers are proportional, the second term is called a *Mean Proportional* between the other two.

The proportion $8 : 4 :: 10 : 5$ may be expressed thus, $\frac{8}{4} = \frac{10}{5}$ (**757**). Reducing these fractions to equivalent ones having a common denominator, $\frac{8 \times 5}{20} = \frac{10 \times 4}{20}$.

Since these fractions are equal, and have a common denominator, their numerators are equal, or $8 \times 5 = 10 \times 4$.

763. PRINCIPLES.—1. *The product of the extremes of a proportion is equal to the product of the means.*

2. *The product of the extremes divided by either mean will give the other mean.*

3. *The product of the means divided by either extreme will give the other extreme.*

EXERCISES.

764. 1. The ratio of 4 to 10 is equal to the ratio of 6 to 15. Express the proportion in all its forms (**757**).

Drill Exercise.—How many terms has a proportion? What are they called? How many ratios? What are they called?

Name the proportionals in example 1; the couplets; the antecedents; the consequents; the extremes; the means. What is the product of the extremes? Of the means? What is the dividend of the first ratio? The divisor of the second ratio? The divisor of the first ratio? The dividend of the second ratio? In the fractional form what are the numerators? The denominators?

2. The ratio of 6 to 15 equals the ratio of 8 to 20.

3. The ratio of $4\frac{1}{2}$ to 18 equals the ratio of 6 to 24.

Change to the form of equations by PRIN. 1:

4. $12 : 1728 :: 1 : 144$.

5. $2\frac{3}{8} : 17 :: 20 : 143\frac{3}{19}$.

6. $27.03 : 9.01 :: 16.05 : 5.35$.

7. $\frac{2}{8} : \frac{3}{7} :: \frac{3}{8} : \frac{9}{14}$.

8. The extremes are 15 and 48, and one of the mean is 10. Find the other mean.

9. The means are 25 and 75, and one of the extrem is $12\frac{1}{2}$. Find the other extreme.

The required or omitted term in a proportion, or in an operation, will hereafter be represented by x.

Find the term omitted in each of the following proportions:

11. $8 : 52 :: 20 : x$.
12. $12 : x :: 1 : 144$.
13. $x : 20 :: 120 : 50$.
14. $\$80 : \$4 :: x : 4$.
15. $2.5 : 62.5 :: 5 : x$.
16. $\$175.35 : \$x :: \frac{1}{8} : \frac{3}{7}$.
17. $4\frac{1}{2}$ yd. : x yd. :: $\$9\frac{3}{4} : \$27.25$.
18. $x : 9.01 :: 16.05 : 5.35$.
19. $\frac{3}{5}$ yd. : x yd. :: $\$\frac{7}{8} : \$59.0625$.
20. $\frac{5}{16} : \frac{3}{8} :: x : \frac{2}{5}$.
21. $x : 38\frac{1}{4} :: 8\frac{1}{2} : 76\frac{1}{2}$.
22. $7.5 : 18 :: x$ oz. : $7\frac{1}{15}$ oz.

SIMPLE PROPORTION.

765. A ***Simple Proportion*** is an expression of equality between two simple ratios. It is used to solve problems of which *three* terms are given, and the *fourth* is required.

Of the three given numbers, two must always be of the same kind; and the *third*, of the same kind as the *required term*.

766. A ***Statement*** is the arrangement of these terms in the form of a proportion.

WRITTEN EXERCISES.

767. 1. If 4 tons of coal cost \$24, what will be the cost of 12 tons at the same rate?

STATEMENT.

$4 \text{ T.} : 12 \text{ T.} :: \$24 : \$x$

OPERATION.

$\overline{12 \times 24} \div 4 = \$72$

Or By Cancellation.

$$\$x = \frac{12 \times \cancel{24}^{6}}{\cancel{4}} = \$72$$

ANALYSIS.—Since 4 tons and 12 tons have the same unit value, they can be compared, and will form one couplet of the proportion.

For the same reason \$24 the cost of 4 tons, and \$$x$ the cost of 12 tons, will form the other couplet.

Then by PRIN. 3, $\$x = 24 \times 12 \div 4 = \$72$.

PROOF.—$4 \times 72 = 12 \times 24$. (**763,** PRIN. 1.) In practice, that number which is of the same unit value as the *required* term, is generally made the antecedent of the second couplet or *third* term of the proportion, and the required term, x, the *fourth* term. The terms of the first couplet are so arranged as to have the same ratio to each other, as the terms of the second couplet, have to each other, which is easily determined by inspection. The product of the means 12 and 24, divided by the given extreme 4, gives the other extreme, or required term, $72. (**763,** PRIN. 3.)

Drill exercises like the following, will soon make the pupil familiar with the principles and operations of proportion.

2. If 4 horses eat 12 bushels of oats in a given time, how many bushels will 20 horses eat in the same time?

In this example, what two numbers have the same unit value? What do they form? What is the denomination of the third term? Of the required term? What is the antecedent of the second couplet? From the conditions of the question, is the consequent of the second couplet or required term, greater or less than the antecedent? If greater, how must the antecedent and consequent of the first couplet compare with each other? If less, how compare? What is the ratio of the first couplet? Why not 20 to 4? Make the statement. How is the required term found?

3. If 96 cords of wood cost $240, what will 40 cords cost?

4. If 20 lb. of sugar cost $1.80, find the cost of 45 lb.

5. If 18 bu. of wheat make 4 barrels of flour, how many barrels will 200 bu. make?

RULE.—I. *Make the statement so that two of the given numbers which are of the same unit value, shall form the first couplet of the proportion, and have a ratio equal to the ratio of the third given term to the required term.*

II. *Divide the product of the means by the given extreme, and the quotient will be the number* required.

CAUSE AND EFFECT.

768. The terms of a proportion have not only the relations of *magnitude*, but also the relations of *cause* and *effect*.

Every problem in proportion may be considered as a comparison of *two causes* and *two effects*.

Thus, if 4 tons as a *cause* will bring when sold, $24 as an *effect*, 12 tons as a *cause* will bring $72 as an *effect*. Or, if 6 horses as a *cause* draw 10 tons as an *effect*, 9 horses as a *cause* will draw 15 tons as an *effect*.

769. Since like causes produce like effects, the ratio of two like causes equals the ratio of two like effects produced by these causes. Hence,

1st cause : 2d cause :: 1st effect : 2d effect.

WRITTEN EXERCISES.

770. 1. If 8 men earn $32 in one week, how much will 15 men earn at the same rate, in the same time?

STATEMENT.

1st cause.		2d cause.		1st effect.		2d effect.
8 men	:	15 men	::	$32	:	$\$x$

OPERATION.

$$\$x = \overline{15 \times 32} \div 8 = \$60$$

ANALYSIS.—In this example an *effect* is required. The first cause is 8 men, the second cause 15 men, and since they are like causes they can be compared.

The effect of the first cause is $32 earned, the effect of the second cause is $\$x$ earned, or the *required* term. Since like effects have the same ratio as their causes (**769**), the causes may form the first couplet, and the effects the second couplet of the proportion. The required term is readily obtained by (**763**, 3).

2. If 20 bushels of wheat produce 6 barrels of flour, *how many* bushels will be required to produce 24 barrels?

STATEMENT.

1st cause.		2d cause.		1st effect.		2d effect.
20 bu.	:	x bu	::	6 bbl.	:	24 bbl.

OPERATION.

$$x \text{ bu.} = \overline{20 \times 24} \div 6 = 80 \text{ bu.}$$

ANALYSIS.—In this example a *cause* is required. The first cause is 20 bu., the second cause is x bu. or the *required* term. The effect of the first cause is 6 bbl. of flour, the effect of the second cause is 24 bbl. of flour. Since like causes have the same ratio as their effects (**769**), the statement is made as in Ex. 1, and the required term found by (**763**, 2).

3. If 5 horses consume 10 tons of hay in 8 mo., how many horses will consume 18 tons in the same time?

Drill Exercise.—In this example, what is the first cause? The second cause? The first effect? The second effect? Is the required term a cause or an effect? A mean or an extreme? What is the first couplet? What, the second? Make the statement. How is the required term found?

4. If 8 yards of cloth cost $6, how many yards can be bought for $75?

5. How many men will be required to build 32 rods of wall in the same time that 5 men can build 10 rods?

RULE.—I. *Arrange the terms in the statement so that the ratio of the causes which form the first couplet, shall equal the ratio of the effects which form the second couplet, putting x in the place of the required term.*

II. *If the required term be an extreme, divide the product of the means by the given extreme; if the required term be a mean, divide the product of the extremes by the given mean.*

To shorten the operation, equal factors in the first and second, or in the first and third terms may be *canceled.*

Solve the following by either of the foregoing methods:

6. If 5 sheep can be bought for $20.75, how many sheep *can be bought for* $398.40?

7. When 10 barrels of flour cost \$112.50, what will be the cost of 476 barrels of flour?

8. If a railroad train run 30 miles in 50 min., in what time will it run 260 miles?

9. How many bushels of peaches can be purchased for \$454.40, if 8 bushels cost \$10.24?

10. If a horse travel 12 miles in 1 hr. 36 min., how far, at the same rate, will he travel in 15 hours?

11. How many days will 12 men require to do a piece of work, that 95 men can do in $7\frac{1}{2}$ days?

12. If $\frac{3}{8}$ of an acre of land cost \$60, what will $45\frac{3}{4}$ acres cost?

13. At the rate of 72 yards for £44 16s., how many yards of cloth can be bought for £5 12s.?

14. If $\frac{7}{8}$ of a barrel of cider cost $\$1\frac{9}{16}$, what is the cost of $\frac{5}{6}$ of a barrel?

15. If the annual rent of 35 A. 90 P. is \$284.50, how much land can be rented for \$374.70?

16. What will 87.5 yd. of cloth cost, if $1\frac{3}{4}$ yd. cost \$1.26?

17. If by selling \$5000 worth of dry goods, a merchant gains \$456.25, what amount must he sell to gain \$1000?

18. Bought coal at \$4.48 per long ton, and sold it at \$7.25 per short ton. What was the gain per ton?

19. What will be the cost of a pile of wood 80 ft. long, 4 ft. wide, 4 ft. high, if a pile 18 ft. long, 4 ft. wide, 6 ft. high cost \$30.24?

20. If 36 bu. of wheat are bought for \$44.50, and sold for \$53.50, what is gained on 480 bu. at the same rate?

21. If a business yield \$700 net profits in 1 yr. 8 mo., in what time will the same business yield \$1050 profits?

COMPOUND PROPORTION.

771. A ***Compound Proportion*** is an expression of equality between two ratios, one or both of which are compound.

All the terms of every problem in compound proportion appear in *couplets*, except *one*, and this is always of the same unit value as the *required* term.

The order of the ratios, and of the terms composing the ratios, is the same as in simple proportion.

WRITTEN EXERCISES.

772. 1. If 18 men build 126 rd. of wall in 60 da., working 10 hr. a day, how many rods will 6 men build in 110 da., working 12 hr. a day?

STATEMENT.

$$\left.\begin{array}{lcl} 18\text{ men} & : & 6\text{ men} \\ 60\text{ days} & : & 110\text{ days} \\ 10\text{ hours} & : & 12\text{ hours} \end{array}\right\} :: \overset{\text{rods.}}{126} : \overset{\text{rods.}}{x}$$

Or,

$$\begin{array}{r|l} x & \cancel{6} \\ 3\ \cancel{18} & \cancel{110}\ 11 \\ 5\ \cancel{60} & \cancel{12} \\ \cancel{10} & \cancel{126}\ 42 \\ \hline 5 & 462 \\ \hline & 92\tfrac{2}{5} = x\text{ rd.} \end{array}$$

OPERATION.

$$\overset{\text{rods.}}{x} = \frac{\cancel{6}\times\overset{11}{\cancel{110}}\times\cancel{12}\times\overset{42}{\cancel{126}}}{\underset{3}{\cancel{18}}\times\underset{5}{\cancel{60}}\times\cancel{10}} = \tfrac{462}{5} = 92\tfrac{2}{5}\text{ rd.}$$

ANALYSIS.—All the terms in this example appear in couplets, except 126 rods, which is of the same unit value as the *required* term, and is made the third term of the proportion, and x rods, the fourth.

The required number of rods depends upon *three* conditions: 1st, the number of men employed; 2d, the number of days they work; and 3d, the number of hours they work each day.

Consider each condition separately, and arrange the terms of the same unit value in couplets, and make the statement as in simple *proportion* (**767**). Then find the required term by (**763**, 3).

2. If 20 horses consume 36 tons of hay in 9 mo., how many tons will 12 horses consume in 18 months?

Drill Exercise—In this example, what is the denomination of the required term? What given number has the same unit value? What will be the third term of the proportion? The fourth? How many couplets are there? Name them. What kind of a ratio do they form? How is the antecedent and consequent of each couplet determined? How is a compound ratio reduced to a simple one? Make the statement. Is the required term a mean or an extreme? How is it found? (**763**, 3.)

3. If $320 will pay the board of 4 persons for 8 weeks, for how many weeks will $800 pay the board of 15 persons?

4. If a man walk 192 miles in 6 days, walking 8 hr. a day, how far can he walk in 18 days, walking 6 hr. a day?

5. If 6 laborers can dig a ditch 34 yards long in 10 days, how many days will 20 laborers require to dig a ditch 170 yards long?

RULE.—I. *Form each couplet of the compound ratio from the numbers given, by comparing those which are of the same unit value, arranging the terms of each in respect to the third term of the proportion, as if it were the first couplet of a simple proportion.* (**767.**)

II. *Divide the product of the second and third terms by the product of the first terms, the quotient will be the number required.*

The same preparation of the terms by reduction is to be observed as in simple proportion.

When possible, shorten the operation by *cancellation*. When the vertical line is used, write the factors of the dividend on the right, and the factors of the divisor with x on the left.

CAUSE AND EFFECT.

773. If we regard the conditions of each problem as the comparison of *two causes* and *two effects*, the compound proportion will consist of two ratios, one or both of which may be compound, and the *required* term will be either a *simple* cause, or effect, or a *single element* of a *compound* cause, or effect.

WRITTEN EXERCISES.

774. 1. If 8 men earn \$320 in 8 days, how much will 12 men earn in 4 days?

STATEMENT.

$$\left.\begin{array}{ll} \text{1st cause.} & \text{2d. cause.} \\ 8 \text{ men} : & 12 \text{ men} \\ 8 \text{ days} \cdot & 4 \text{ days} \end{array}\right\} :: \$320 : \$x$$

OPERATION.

$$\$x = \frac{12 \times 4 \times \overset{5}{\cancel{320}}}{\cancel{8} \times \cancel{8}} = \$240$$

Or,

$$\begin{array}{r|r} \$x & 12 \\ \cancel{8} & 4 \\ \cancel{8} & \$\overset{5}{\cancel{320}} \\ \hline \$x & \$240 \end{array}$$

ANALYSIS. — In this example the first cause is 8 men at work 8 days, the second cause is 12 men at work 4 days; the two form a compound ratio.

The effect of the first cause is \$320 earned, the effect of the second cause is \$$x$ earned, and is the *required* term; the two effects form a simple ratio.

The value of the *required* term depends upon *two* conditions: 1st, the number of men at work; 2d, the number of days they work. Consider each condition separately, and arrange the terms of the same unit value in couplets, and make a statement in the same manner as in simple proportion. The required term being an *extreme*, is found by (**763,** 3).

2. If it cost \$41.25 to pave a sidewalk 5 ft. wide and 75 ft. long, what will it cost to pave a similar walk 8 ft. wide and 566 ft. long?

3. How many days will 21 men require to dig a ditch 80 ft. long, 3 ft. wide, and 8 ft. deep, if 7 men can dig a ditch 60 ft. long, 8 ft. wide, and 6 ft. deep, in 12 days?

STATEMENT.

$$\left.\begin{array}{r} 7 : 21 \\ 12 : \ x \end{array}\right\} :: \left\{\begin{array}{r} 60 : 80 \\ 8 : \ 3 \\ 6 : \ 8 \end{array}\right.$$

OPERATION.

$$x = \frac{\cancel{7} \times \cancel{12} \times \overset{8}{\cancel{80}} \times \cancel{3} \times \cancel{8}}{\cancel{21} \times \underset{3}{\cancel{60}} \times \cancel{8} \times \cancel{6}} = \frac{8}{3} = 2\tfrac{2}{3} \text{ da.}$$

Or,

$$\begin{array}{rr|ll} & x & \cancel{7} & \\ & \cancel{21} & \cancel{12} & \\ 3 & \cancel{60} & \cancel{80} & 8 \\ & \cancel{8} & \cancel{3} & \\ & \cancel{6} & \cancel{8} & \\ \hline & 3 & 8 & \end{array}$$

$$x = 2\tfrac{2}{3} \text{ da.}$$

Analysis.—In this example the causes and the effects each form a compound ratio. The *required* term is an element of the second cause and a *mean*. Hence divide the product of the extremes by the product of the given means, and the quotient is the required factor or term, 2⅔ da. (**763,** 2).

4. If 4 horses consume 48 bushels of oats in 12 days, how many bushels will 20 horses consume in 8 weeks?

Rule.—I. *Of the given numbers, select those which constitute the causes, and those which constitute the effects, and arrange them in couplets as in simple cause and effect, putting x in the place of the required term.*

II. *If the required term, x, be an extreme, divide the product of the means by the product of the given extremes; if x be a mean, divide the product of the extremes by the product of the given means; the quotient will be the required term.*

Solve the following by either of the foregoing methods:

5. What sum of money will produce \$300 in 8 mo., if \$800 produce \$70 in 15 months?

6. If 20 reams of paper are required to print 800 copies of a book containing 230 pages each, 40 lines on a page, how many reams are required to print 3000 copies of 400 pages each, 35 lines on a page?

7. If 10 men can cut 46 cords of wood in 18 da., working 10 hr. a day, how many cords can 40 men cut in 24 da., working 9 hr. a day?

8. What is the cost 36½ yards of cloth 1½ yd. wide, if 2½ yards 1⅖ yd. wide, cost $3.37½?

9. A contractor employs 45 men to complete a work in 3 months; what additional number of men must he employ, to finish the work in 2½ months?

10. If a vat 16 ft. long, 7 ft. wide, and 15 ft. deep holds 384 barrels, how many barrels will a vat 17½ ft. long, 10½ ft. wide, and 13 ft. deep hold?

11. What is the weight of a block of granite 8 ft. long, 4 ft. wide, and 10 in. thick, if a similar block 10 ft. long, 5 ft. wide, and 16 in. thick, weigh 5200 pounds?

12. If it cost $15 to carry 20 tons 1½ miles, what will it cost to carry 400 tons ½ of a mile?

13. If it take 13500 bricks to build a wall 200 ft. long, 20 ft. high, and 16 in. thick, each brick being 8 in. long, 4 in. wide, and 2 in. thick, how many bricks 10 in. long, 5 in. wide, 3½ in. thick, will be required to build a wall 600 ft. long, 24 ft. high, and 20 ft. thick?

14. What will 15 hogsheads of molasses cost, if 28½ gallons cost $7⅛?

15. At 6¼d. for 1¾ yards of cotton cloth, how many yards can be bought for £10 6s. 8d.?

16. If $750 gain $202.50 in 4 yr. 6 mo., what sum will gain $155.52 in 1 yr. 6 mo.?

17. In what time can 60 men do a piece of work that 15 men can do in 20 days?

18. If $2\frac{1}{2}$ yd. of cloth 6 quarters wide can be made from 1 lb. 12 oz. of wool, how many yards of cloth 4 quarters wide can be made from 70 lb. of wool?

19. If the use of $300 for 1 yr. 8 mo. is worth $30, how long, at the same rate, may $210.25 be retained to be worth $42.891?

20. A farmer has hay worth $18 a ton, and a merchant has flour worth $10 a barrel. If the farmer ask $21 for his hay, what should the merchant ask for his flour?

21. How many men will be required to dig a cellar 45 ft. long, 34.6 ft. wide, and 12.3 ft. deep, in 12 da. of 8.2 hr. each, if 6 men can dig a similar one 22.5 ft. long, 17.3 ft. wide, and 10.25 ft. deep, in 3 da. of 10.25 hr. each?

22. If a bin 8 ft. long, $4\frac{1}{5}$ ft. wide, and $2\frac{1}{2}$ ft. deep, hold $67\frac{1}{2}$ bu., how deep must another bin be made, that is 18 ft. long and $3\frac{5}{6}$ ft. wide, to hold 450 bu.?

23. What will 120 lb. of coffee cost, if 10 lb. of sugar cost $1.25, and 16 lb. of sugar are worth 5 lb. of coffee?

24. Two men have each a farm. A's farm is worth $48.75, and B's $$43\frac{1}{8}$; but in trading A values his at $60 an acre. What value should B put upon his?

25. If 6 men in 4 mo., working 26 da. for a month, and 12 hr. a day, can set the type for 24 books of 300 pp. each, 60 lines to the page, 12 words to the line, and an average of 6 letters to the word, in how many months of 24 da. each, and 10 hr. a day, can 8 men and 4 boys set the type for 10 books of 240 pp. each, 52 lines to the page, 16 words to the line, and 8 letters to the word, 2 *boys doing as* much as 1 man?

PARTNERSHIP

ORAL EXERCISES.

775. 1. If John has 10 marbles, William 15 marbles, and Charles 25 marbles, what part of the whole has each?

2. Two men bought a barrel of flour for $9, the first paying $4 and the second $5. What part of the flour belongs to each?

3. Three men bought 108 sheep, and as often as the first paid $3, the second paid $4, and the third $5. How many sheep should each receive?

4. If $45 be divided between two persons, so that of every $5, one receives $2, and the other $3, how many dollars does each receive?

5. Two men hired a pasture for $36; one put in 2 horses for 3 weeks, the other 3 horses for 4 weeks. What should each pay?

DEFINITIONS.

776. ***Partnership*** is the association of two or more persons under a certain name, for the transaction of business with an agreement to share the gains and losses.

777. A ***Firm, Company*** or ***House*** is any particular partnership association.

778. The ***Capital*** is the money or property invested by the partners, called also *Investment*, or Joint-Stock.

779. The ***Resources*** of a firm are the amounts due it, together with the property of all kinds belonging to it; called also *Assets*, or *Effects*.

780. The ***Liabilities*** of a firm are its *debts*.

781. The ***Net Capital*** is the excess of resources over liabilities.

WRITTEN EXERCISES.

782. To apportion gains or losses according to capital invested.

1. A and B engage in trade; A furnishes $400 capital, B $600. They gain $250; what is the profit of each?

1ST OPERATION. (*By Fractions.*)

$400, A.'s investment $= \frac{400}{1000} = \frac{2}{5}$ of the whole capital.
600, B.'s " $= \frac{600}{1000} = \frac{3}{5}$ " " "
$1000, whole "
$250 × $\frac{2}{5}$ = $100, A.'s share of the gain.
$250 × $\frac{3}{5}$ = $150, B.'s " " "

2D OPERATION. (*By Proportion.*)

$1000 (whole cap.) : $400 (A.'s inv.) : : $250 (whole gain) : A.'s share.
$1000 (whole cap.) : $600 (B.'s inv.) : : $250 (whole gain) : B.'s share.

3D OPERATION. (*By Percentage.*)

$250 gain is $\frac{250}{1000} = 25\%$ of the whole capital.
$400 × .25 = $100, A.'s gain; $600 × .25 = $150, B.'s gain.

ANALYSIS.—(*1st Method.*) Since $400, A.'s investment, is $\frac{400}{1000}$, or $\frac{2}{5}$, of the whole capital, he is entitled to $\frac{2}{5}$ of the gain, or $100; and B is entitled to $\frac{3}{5}$ of the gain, or $150.

2d Method. The ratio of $1000, the whole capital, to $400, A.'s investment, is equal to the ratio of $250, the whole gain, to A.'s share of the gain. Hence the proportions, etc.

3d Method. Since the gain is 25% of the whole capital, each partner is entitled to 25% of his investment as his share of the gain.

The *third method* (by dividend) is that generally adopted by *joint-stock companies* having numerous shareholders.

2. At the end of the year, Norton, Smith & Co. take an account of stock, and find the amount of merchandise, as per inventory, to be $8400; cash on hand, $4850; due from sundry persons, $5273. Their debts are found to amount to $4223. S. Norton's investment in the business is $5000; R. Smith's, $4000; and C. Woodward's, $2000. Make a statement showing the resources, liabilities, net capital, and net gain; and find each partner's share of the gain.

OPERATION.

Resources.

Mdse. as per inventory,	$8400	
Cash on hand,	4850	
Debts due the firm,	5273	
		$18523

Liabilities.

Debts due to sundry persons,	4223
Net capital,	$14300

Investments.

S. Norton,	$5000	
R. Smith,	4000	
C. Woodward,	2000	
Total investments		$11000
Net gain,		$3300

S. Norton's fractional part, $\frac{5000}{11000} = \frac{5}{11}$ of $3300 = $1500.
R. Smith's " " $\frac{4000}{11000} = \frac{4}{11}$ of $3300 = $1200.
C. Woodward's " " $\frac{2000}{11000} = \frac{2}{11}$ of $3300 = $ 600.

PROOF.—$1500 + $1200 + $600 = $3300, total gain.

RULE 1. *Find what fractional part each partner's investment is of the whole capital, and take such part of the whole gain or loss for his share of the gain or loss.* Or,

2. *State by proportion, as the whole capital is to each partner's investment, respectively, so is the whole gain or loss to each partner's share of the gain or loss.* Or,

3. *Find what per cent. the gain or loss is of the whole capital, and take that per cent. of each partner's investment for his share of the gain or loss, respectively.*

3. A furnishes $4000, B, $2700, and C, $2300, to purchase a house, which they rent for $720. What is each one's share of the rent?

4. Four persons rent a farm of 230 A. 64 P. at $7½ an acre. A puts in 288 sheep, B, 320 sheep, C, 384 sheep, and D, 648 sheep; what rent ought each to pay?

5. Prime & Co. fail in business; their liabilities amount to $22000; their available resources to $8800. They owe A $4275, and B $2175.50: what will each of these creditors receive?

6. Four persons engage in manufacturing, and invest jointly $22500. At the expiration of a certain time, A's share of the gain is $2000, B's $2800.75, C's $1685.25, and D's $1014. How much capital did each put in?

7. An estate worth $10927.60 is divided between two heirs so that one receives ⅛ more than the other. What does each receive?

8. Three persons engage in the lumber trade with a joint capital of $37680. A puts in $6 as often as B puts in 10, and as often as C puts in $14. Their annual gain *is equal to C's* stock. What is each partner's gain?

9. Ames, Lyon & Co. close business in the following condition: notes due the firm to the amount of \$24843.75, cash in hand, \$42375.80, due on account, \$26500, merchandise per inventory, \$175840. Notes against the firm, \$14058.75, due from the firm on account, \$12375.80. Ames invested \$60000, Lyon, \$40000, and Clark \$25000. Make a statement showing the total amount of resources, liabilities, investments, net capital, net gain, and each partner's share of the gain.

783. To apportion gains or losses according to amount of capital invested, and time it is employed.

1. Three partners, A, B, and C, furnish capital as follows: A, \$500 for 2 mo.; B, \$400 for 3 mo.; C, \$200 for 4 mo. They gain \$600; what is each partner's share?

OPERATION.

$$\left.\begin{array}{l} 500 \times 2 = 1000 = \frac{1000}{3000} = \frac{1}{3} \times \\ 400 \times 3 = 1200 = \frac{1200}{3000} = \frac{2}{5} \times \\ 200 \times 4 = \underline{\ 800\ } = \frac{800}{3000} = \frac{4}{15} \times \end{array}\right\} \$600 = \left\{\begin{array}{l} \$200,\ \text{A's share.} \\ \$240,\ \text{B's} \quad \text{“} \\ \$160,\ \text{C's} \quad \text{“} \end{array}\right.$$
$$\ 3000$$

Analysis.—The use of \$500 for 2 mo. is the same as the use of \$1000 for 1 mo.; the use of \$400 for 3 mo. is the same as that of \$1200 for 1 mo.; and the use of \$200 for 4 mo. is the same as that of \$800 for 1 mo. Therefore the whole capital is the use of \$3000 for 1 mo.; and as A's investment is \$1000 for 1 mo., it is $\frac{1}{3}$ of the capital, and hence he should receive $\frac{1}{3}$ of the gain, or \$200. For the same reason, B should receive $\frac{2}{5}$, and C $\frac{4}{15}$ of the gain, or \$240 and \$160, respectively.

The other methods of operation may be applied by considering the products of investment and time as shares of the capital. Thus, \$600 is 20% of \$3000; and 20% of \$1000, \$1200, and \$800 will give *\$200, \$240, and \$160,* respectively, the shares of gain required.

2. Barr, Banks & Co. gain in trade $8000. Barr furnished $12000 for 6 mo., Banks, $10000 for 8 mo., and Butts $8000 for 11 mo. Apportion the gain?

RULE 1.—*Multiply each partner's capital by the time it is invested, and divide the whole gain or loss among the partners in the ratio of these products.* Or,

2. *State by proportion: The sum of the products is to each product, as the whole gain or loss is to each partner's gain or loss.*

3. Jan. 1, 1876, three persons began business with $1300 capital furnished by A; March 1, B put in $1000; Aug. 1, C put in $900. The profits at the end of the year were $750. Apportion it.

4. In a partnership for 2 years, A furnished at first $2000, and 10 mo. after withdrew $400 for 4 mo., and then returned it; B at first put in $3000, and at the end of 4 mo. $500 more, but drew out $1500 at the end of 16 mo. The whole gain was $3372. Find the share of each.

5. The joint capital of a company was $5400, which was doubled at the end of the year. A put in $\frac{1}{2}$ for 9 mo., B $\frac{2}{5}$ for 6 mo., and C the remainder for 1 year. What is each one's share of the stock at the end of the year?

6. Crane, Child & Co. forming a partnership Jan. 1, 1875, invested and drew out as follows: Crane invested $2000, 4 mo. after $1000 more, and at the end of 9 mo. drew out $600. Child invested $5000, 6 mo. after $1200 more, and at the end of 11 mo. put in $2000 more. Coe put in $6000, 4 mo. after drew out $4000, and at the end of 8 mo. drew out $1000 more. The net profits for the year were $7570. Find the share of each.

ALLIGATION

784. ***Alligation*** treats of mixing or compounding two or more ingredients of different values or qualities.

785. ***Alligation Medial*** is the process of finding the *mean* or *average* value or quality of several ingredients.

786. ***Alligation Alternate*** is the process of finding the proportional quantities to be used in any required mixture.

WRITTEN EXAMPLES.

787. 1. If a grocer mix 8 lb. of tea worth \$.60 a pound with 6 lb. at \$.70, 2 lb. at \$1.10, and 4 lb. at \$1.20, what is 1 lb. of the mixture worth?

OPERATION.

$$\begin{array}{rcr} \$.60 \times 8 = & & \$4.80 \\ .70 \times 6 = & & 4.20 \\ 1.10 \times 2 = & & 2.20 \\ 1.20 \times \underline{4} = & & \underline{4.80} \\ 20 &) & \underline{\$16.00} \end{array}$$

ANALYSIS.—Since 8 lb. of tea at \$.60 is worth \$4.80, and 6 lb. at \$.70 is worth \$4.20, and 2 lb. at \$1.10 is worth \$2.20, and 4 lb. at \$1.20 is worth \$4.80, the mixture of 20 lb. is worth \$16. Hence 1 lb. is worth $\frac{1}{20}$ of \$16, or \$16 ÷ 20 = \$.80.

2. If 20 lb. of sugar at 8 cents be mixed with 24 lb. at 9 cents, and 32 lb. at 11 cents, and the mixture is sold at 10 cents a pound, what is the gain or loss on the whole?

RULE.—*Find the entire cost or value of the ingredients, and divide it by the sum of the simples.*

3. A miller mixes 18 bu. of wheat at $1.44 with 6 bu. at $1.32, 6 bu. at $1.08, and 12 bu. at $.84. What will be his gain per bushel if he sell the mixture at $1.50?

4. Bought 24 cheeses, each weighing 25 lb., at 7¢ a pound; 10, weighing 40 lb. each, at 10¢; and 4, weighing 50 lb. each, at 13¢; sold the whole at an average price of 9½¢ a pound. What was the whole gain?

5. A drover bought 84 sheep at $5 a head; 96 at $4.75; and 130 at $5½. At what average price per head must he sell them to gain 20%?

788. To find the proportional parts to be used, when the mean price of a mixture and the prices of the simples are given.

1. What relative quantities of timothy seed worth $2 a bushel, and clover seed worth $7 a bushel, must be used to form a mixture worth $5 a bushel?

OPERATION.

$$5\left\{\begin{array}{c|c|c}2 & \frac{1}{3} & 2\\ 7 & \frac{1}{2} & 3\end{array}\right\}\textit{Ans.}$$

ANALYSIS.—Since on every ingredient used whose price or quality is *less* than the mean rate there will be a *gain*, and on every ingredient whose price or quality is *greater* than the mean rate there will be a *loss*, and since the gains and losses must be exactly equal, the relative quantities used of each should be such as represent the unit of *value*. By selling one bushel of timothy seed worth $2, for $5, there is a gain of $3; and to gain $1 would require ⅓ of a bushel, which is placed opposite the 2. By selling one bushel of clover seed worth $7, for $5, there is a loss of $2; and to lose $1 would require ½ of a bushel, which is placed opposite the 7.

In every case, to find the unit of value, divide $1 by the gain or loss per bushel or pound, etc. Hence, if every time ⅓ of a bushel of timothy seed is taken, ½ of a bushel of clover seed is taken, the gain and loss will be exactly equal, and ⅓ and ½ will be the *proportional quantities* required.

To express the proportional numbers in integers, reduce these fractions to a common denominator, and use their numerators, since fractions having a common denominator are to each other as their numerators (**241**); thus, $\frac{1}{3}$ and $\frac{1}{2}$ are equal to $\frac{2}{6}$ and $\frac{3}{6}$, and the proportional quantities are 2 bu. of timothy seed to 3 bu. of clover seed.

2. What proportions of teas worth respectively 3, 4, 7, and 10 shillings a pound, must be taken to form a mixture worth 6 shillings a pound?

OPERATION.

		1	2	3	4	5
6	3	$\frac{1}{3}$		4		4
	4		$\frac{1}{2}$		1	1
	7		1		2	2
	10	$\frac{1}{4}$		3		3

ANALYSIS.—To preserve the equality of gains and losses, always compare two prices or simples, one *greater* and one *less* than the mean rate, and treat each pair or couplet as a separate example. In the given example form two couplets, and compare either 3 and 10, 4 and 7, or 3 and 7, 4 and 10.

We find that $\frac{1}{3}$ of a lb. at 3s. must be taken to *gain* 1 shilling, and $\frac{1}{4}$ of a lb. at 10s. to *lose* 1 shilling; also $\frac{1}{2}$ of a lb. at 4s. to *gain* 1 shilling, and 1 lb. at 7s. to *lose* 1 shilling. These proportional numbers, obtained by comparing the two couplets, are placed in columns 1 and 2. If, now, the numbers in columns 1 and 2 are reduced to a common denominator, and their numerators used, the integral numbers in columns 3 and 4 are obtained, which, being arranged in column 5, give the *proportional quantities* to be taken of each.

It will be seen that in comparing the simples of any couplet, one of which is greater, and the other less than the mean rate, the proportional number finally obtained for either term is the difference between the mean rate and the other term. Thus, in comparing 3 and 10, the proportional number of the former is 4, which is the difference between 10 and the mean rate 6; and the proportional number of the latter is 3, which is the difference between 3 and the mean rate. The same is true of every other couplet. Hence, when the simples and the mean rate are integers, the intermediate steps taken to obtain the final proportional numbers as in columns 1, 2, 3, and 4, may be omitted, and the same results readily found by taking the difference between each simple and the mean rate, and placing it opposite the one with which it is compared.

3. In what proportions must sugars worth 10 cents, 11 cents, and 14 cents a pound be used, to form a mixture worth 12 cents a pound?

4. A farmer has sheep worth \$4, \$5, \$6, and \$8 per head. What number may he sell of each and realize an average price of \$5½ per head?

RULE.—I. *Write the several prices or qualities in a column, and the mean price or quality of the mixture at the left.*

II. *Form couplets by comparing any price or quality less, with one that is greater than the mean rate, placing the part which must be used to gain* 1 *of the mean rate opposite the less simple, and the part that must be used to lose* 1 *opposite the greater simple, and do the same for each simple in every couplet.*

III. *If the proportional numbers are fractional, they may be reduced to integers, and if two or more stand in the same horizontal line, they must be added; the final results will be the proportional quantities required.*

1. If the numbers in any couplet or column have a common factor, it may be rejected.

2. We may also multiply the numbers in any couplet or column by any multiplier we choose, without affecting the equality of the gains and losses, and thus obtain an indefinite number of results, any one of which being taken will give a correct final result.

5. What amount of flour worth \$5½, \$6, and \$7¾ per barrel, must be sold to realize an average price of \$6¼ per barrel?

6. In what proportions can wine worth \$1.20, \$1.80, and \$2.30 per gallon be mixed with water so as to form a *mixture worth* \$1.50 per gallon?

789. When the quantity of one of the simples is limited.

1. A farmer has oats worth \$.30, corn worth \$.45, and barley worth \$.84 a bushel. To make a mixture worth \$.60 a bushel, and which shall contain 48 bu. of corn, how many bushels of oats and barley must he use?

OPERATION.

$$60\left\{\begin{array}{r|c|c|c|c|r|r} 30 & \frac{1}{30} & & 4 & & 4 & 24 \\ 45 & & \frac{1}{15} & & 8 & 8 & 48 \\ 84 & \frac{1}{24} & \frac{1}{24} & 5 & 5 & 10 & 60 \end{array}\right\}\textit{Ans.}$$

ANALYSIS. — By the same process as in (**788**), the proportional quantities of each are found to be 4 bu. of oats, 8 of corn, and 10 of barley. But since 48 bu. of corn is 6 times the proportional number 8, to preserve the equality of gain and loss take 6 times the proportional quantity of each of the other simples, or $6 \times 4 = 24$ bu. of oats, and $6 \times 10 = 60$ bu. of barley. Hence, etc.

2. A dairyman bought 10 cows at \$40 a head. How many must he buy at \$32, \$36, and \$48 a head, so that the whole may average \$44 a head?

RULE.—*Find the proportional quantities as in* (**788**). *Divide the given quantity by the proportional quantity of the same ingredient, and multiply each of the other proportional quantities by the quotient thus obtained.*

3. A grocer having teas worth \$.80, \$1.20, \$1.50, and \$1.80 per pound, wishes to form a mixture worth \$1.60 a pound, and use 20 lb. of that worth \$1.50 a pound.

4. Bought 12 yd. of cloth for \$30. How many yards must I buy at \$3½ and \$1½ a yard, that the average price of the whole may be \$2⅖ a yard?

5. How many acres of land worth \$70 an acre must be added to a farm of 75 A., worth \$100 an acre, that the average value may be \$80 an acre?

790. When the quantity of the whole compound is limited.

1. A grocer has sugars worth 6 cents, 7 cents, 12 cents, and 13 cents per pound. He wishes to make a mixture of 108 pounds, worth 10 cents a pound; how many pounds of each kind must be use?

OPERATION.

10	6	$\frac{1}{4}$		3		3	27
	7		$\frac{1}{3}$		2	2	18
	12		$\frac{1}{2}$		3	3	27
	13	$\frac{1}{3}$		4		4	36
						12	120

ANALYSIS.—The proportional quantities of each simple found by (**788**) are 3 lb. at 6 cts., 2 lb. at 7 cts., 3 lb. at 12 cts., and 4 lb. at 13 cts. Adding the proportional quantities, the mixture is but 12 lb., while the required mixture is 108, or 9 times 12. If the whole mixture is to be 9 times as much as the *sum* of the proportional quantities, then the quantity of each simple used must be 9 times as much as its respective proportional, or 27 lb. at 6 cts., 18 lb. at 7 cts., 27 lb. at 12 cts., and 36 lb. at 13 cts.

2. A man paid $330 to 55 laborers, consisting of men, women, and boys; to the men he paid $10 a week, to the women $2 a week, and to the boys $1 a week; how many were there of each?

RULE.—*Find the proportional numbers as in* (**788**). *Divide the given quantity by the sum of the proportional quantities, and multiply each of the proportional quantities by the quotient thus obtained.*

3. How much water must be mixed with wine worth $.90 a gallon, to make 100 gal. worth $.60 a gallon?

4. One man and 3 boys received $84 for 56 days' labor; the man received $3 per day, and the boys $$\frac{1}{2}$, $$\frac{3}{4}$, and $$1\frac{3}{4}$ *respectively*; how many days did each labor?

791. SYNOPSIS FOR REVIEW.

- RATIO.
  - 1. Defs.
    - 1. Ratio. 2. Sign of Ratio. 3. Terms. 4. Antecedent. 5. Consequent. 6. Value of a Ratio. 7. Simple Ratio. 8. Compound Ratio. 9. Reciprocal of a Ratio.
  - 2. Formulas, 1, 2, 3.
  - 3. General Principles, 1, 2, 3.
  - 4. General Law.
- PROPORTION.
  - 1. Defs.
    - 1. Proportion. 2. Sign. 3. Couplet. 4. Proportional. 5. Antecedents. 6. Consequents. 7. Extremes. 8. Means. 9. Mean Proportional.
  - 2. Principles, 1, 2, 3, 4.
  - 3. Simple Proportion.
    - 1. Defs.
      - 1. *Simple Proportion.*
      - 2. *Statement.*
    - 2. Rule, I, II.
    - 3. Cause and Effect.
    - 4. Rule, I, II.
  - 4. Compound Proportion.
    - 1. Def. Compound Proportion.
    - 2. Rule, I, II.
    - 3. Cause and Effect.
    - 4. Rule, I, II.
- PARTNERSHIP.
  - 1. Defs.
    - 1. Partnership. 2. Firm, Company, or House. 3. Capital. 4. Resources. 5. Liabilities. 6. Net Capital.
  - 2. **782.** Rule, 1, 2, 3.
  - 3. **783.** Rule, 1, 2.
- ALLIGATION.
  - 1. Defs.
    - 1. Alligation. 2. Alligation Medial. 3. Alligation Alternate.
  - 2. **787.** Rule.
  - 3. **788.** Rule, I, II, III.
  - 4. **789.** Rule.
  - 5. **790.** Rule.

TEST PROBLEMS.

792. 1. The sum of two numbers is 120, and their difference is equal to $\frac{1}{3}$ of the greater. Find the numbers.

2. E's age is $1\frac{1}{2}$ times the age of D, and F's age is $2\frac{1}{10}$ times the age of both, and the sum of their ages is 124. What is the age of each?

3. If 7 bu. of wheat are worth 10 bu. of rye, and 5 bu. of rye are worth 14 bu. of oats, and 6 bu. of oats are worth $6, how many bushels of wheat will $60 buy?

4. A mechanic was engaged to labor 20 days, on the conditions that he was to receive $5 a day for every day he worked, and to forfeit $2 a day for every day he was idle; at the end of the time he received $86. How many days did he labor?

5. One man can build a fence in 18 da., working 10 hr. a day; another can build it in 9 da., working 8 hr. a day. In how many days can both together build it, if they work 6 hours a day?

6. If 6 boxes of starch and 7 boxes of soap cost $33, and 12 boxes of starch and 10 boxes of soap cost $54, what is the price of 1 box of each?

7. Three men agree to build a barn for $540. The first and second can do the work in 16 da., the second and third in $13\frac{1}{3}$ da., and the first and third in $11\frac{3}{7}$ da. In how many days can all do it working together? In how many days can each do it alone? What part of the pay should each receive?

8. A dealer paid $182 for 20 barrels of flour, giving $10 for first quality, and $7 for second quality. How many *barrels were* there of each?

9. The hour and minute hands of a clock are together at 12 M. When will they be exactly together the third time after this?

10. Bought 15 bu. of wheat and 30 bu. of oats for $35, and 9 bu. of wheat and 6 bu. of oats for $15. What was the price per bushel of each?

11. If Ames can do as much work in 3 days as Jones can do in 4½ days, and Jones can do as much in 9 days as Smith can do in 12 days, and Smith as much in 10 days as Ray in 8 days, how many days' work done by Ray are equal to 5 days done by Ames?

12. A merchant bought 40 pieces of cloth, each piece containing 25 yd., at $4⅜ per yard, on 9 mo. credit, and sold the same at $4⅝ per yard, on 4 mo. credit. Find his net cash gain, money being worth 6%.

13. There are 70 bu. of grain in 2 bins, and in 1 bin are 10 bu. less than ⅗ as much as there is in the other. How many bushels in the larger bin?

14. Three men can perform a piece of work in 12 hr.; A and B can do it in 16 hr., A and C in 18 hr. What part of the work can B and C do in 9⅐ hours?

15. What per cent. in advance of the cost must a merchant mark his goods, so that after allowing 5% of his sales for bad debts, an average credit of 6 mo., and 4% of the cost of the goods for his expenses, he may make a clear gain of 12½% on the first cost of the goods, money being worth 7%?

16. An elder brother's fortune is 1½ times his younger brother's; the interest of ½ of the elder brother's fortune and ⅓ of the younger's for 5 years, at 6%, is $2400. What is *the fortune of each?*

17. The top of Trinity Church steeple in New York is 268 ft. from the ground; $\frac{3}{4}$ the height of the steeple above the church plus 12 ft. is equal to the height of the church. Find the height of the steeple above the church?

18. Two persons have the same income: A saves $\frac{1}{4}$ of his, but B by spending $300 a year more than A, at the end of 2 years is $200 in debt. What is their income?

19. Divide $2520 among 3 persons, so that the second shall have $\frac{3}{4}$ as much as the first, and the third $\frac{1}{2}$ as much as the other two. What is the share of each?

20. A man owes a debt to be paid in 4 equal installments of 4, 9, 12, and 20 months respectively; a discount of 5% being allowed, he finds that $1500 ready money will pay the debt. What is the amount of the debt?

21. A quantity of flour is to be distributed among some poor families; if 50 lb. are given to each family, there will be 6 lb. left; if 51 lb. are given to each, there will be wanting 4 lb. What is the quantity of flour?

22. I have three notes payable as follows: one for $400, due Jan. 1, 1875; another for $700, due Sept. 1, 1875; and another for $1000, due April 1, 1876. What is the average of maturity?

23. An estate worth $123251.82 is left to four sons, whose ages are 19, 17, 13, and 11 years, respectively, and is to be so divided that each part being put out at 7% simple interest, the amounts shall be equal when they become 21 years of age. What are the parts?

24. If a piece of silk cost $1.20 a yard, at what price must it be marked that it may be sold at 10% less than the marked price, and still make a profit of 20%?

25. A farmer sold 100 geese and turkeys; for the geese

he received $.75 apiece, and for the turkeys $1.25 apiece, and for the whole $104. What was the number of each?

26. A man left his property to three sons; to A ⅓ wanting $180, to B ¼, and to C the rest, which was $590 less than A and B received. What was the whole estate?

27. What is the simple interest and the amount; the compound interest and amount; the present worth and the true discount; the bank discount and the proceeds of $1920, for 2 yr. 5 mo. 12 da., at 6%? Also the face of the note, which when discounted at a bank for the same time, and at the same rate, will produce the same sum?

28. Divide $1500 among 3 persons, so that the share of the second may be ½ greater than that of the first, and the share of the third ½ greater than that of the second.

29. A merchant owes for three bills of goods as follows: $500 due March 1, $800 due June 1, and $600 due Aug. 1. He wishes to give two notes for the amount, one for $1000, payable April 1; what must be the face, and when the maturity, of the other?

30. A man in New York purchased a draft on Chicago for $10640, drawn at 60 da., $10283.56. What was the course of exchange?

31. B. B. Northrop, through his broker, invested a certain sum in U. S. 6's, 5-20, at 107½, and twice as much in U. S. 5's of '81, at 98½, brokerage on each, ¼%. His income from both investments is $1674. How much did he invest in each kind of stock?

32. A, B, and C are under a joint contract to furnish 6000 bu. of corn, at $.48 a bushel; A's corn is worth $.45, B's $.51, and C's $.54; how many bushels must each put into the mixture that the contract may be fulfilled?

33. A cask contains 42¾ U. S. gallons of wine, worth \$4½ per gallon. How much less will it cost in U. S. money, at the rate of £1 2s. per the Imperial gallon?

34. A garden 400 ft. long and 300 ft. wide has a walk 20 ft. wide laid off from it on each side. What is the ratio between the area of the walk and the area of what remains?

35. A commission merchant in Charleston received into his store on May 1, 1875, 1000 bbl. of flour, paying as charges on the same day, freight \$175.48, cartage \$56.25, and cooperage \$8.37. He sold out the shipment as follows: June 3, 200 bbl. @ \$6.25; June 30, 350 bbl. @ \$6.50: July 29, 400 bbl. @ \$6.12½; Aug. 6, 50 bbl. @ \$6. Required, the net proceeds, and the date when they shall be accredited to the owner, allowing commission at 3½%, and storage at 2 cents per week per bbl.

36. Three men engage in manufacturing. L puts in \$3840 for 6 mo.; M, a sum not specified for 12 mo.; and N, \$2560 for a time not specified. L received \$4800 for his capital and profits; M, \$9600 for his; and N, \$4160 for his. Required, M's capital and N's time.

37. My expenditures in building a house, in the year 1874, were as follows: Jan. 16, \$536.78; Feb. 20, \$425.36; March 4, \$259.25; April 24, \$786.36. At the last date I sold the house for exactly what it cost, interest at 6 per cent. on the money expended added, and took the purchaser's note for the amount. What was the face of the note?

38. A man bought a farm for \$6000, and agreed to pay principal and interest in 3 equal annual installments. What was the annual payment, interest being 6%?

INVOLUTION

ORAL EXERCISES.

793. 1. What is the product of 3 used twice as a factor?

2. What is the product of 3 used 3 times as factor .

3. What is the product of 4 used 3 times as a factor?

4. What is the result of using 5 twice as a factor?

5. What is the product of $\frac{1}{2}$ used twice as a factor?

6. What is the result of using $\frac{3}{4}$ twice as a factor? $\frac{2}{3}$, three times as a factor?

7. What number will be produced by using .3 twice as a factor? .7, twice? .4, three times? .05, twice?

8. A room is 9 ft. on each side; how many square feet in the floor?

9. A cubical block of stone is 4 ft. on each edge; how many cubic feet does it contain?

DEFINITIONS.

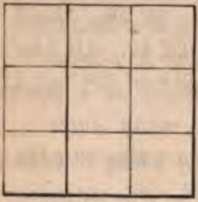
794. A ***Power*** of a number is the product of factors, each of which is equal to that number. Thus, 27 is the third power of 3, since $27 = 3 \times 3 \times 3$.

795. ***Involution*** is the process of finding any power of a number.

796. The ***Base*** or ***Root*** of a power is one of the equal factors of the power. Thus, 27 is the third power of 3, and 3 is the *base*, or *root*, of that power.

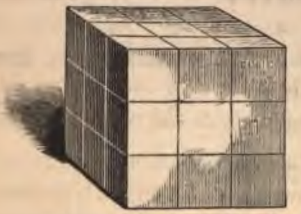
797. The ***Exponent*** of a power is a number placed at the right of the base and a little above it, to show how many times it is used as a factor to produce the power. It also denotes the *degree* of the power. Thus,

3^1 or $3 = 3$, the *first* power of 3.
$3^2 = 3 \times 3 = 9$, the *second* power of 3.
$3^3 = 3 \times 3 \times 3 = 27$, the *third* power of 3.
$3^4 = 3 \times 3 \times 3 \times 3 = 81$, the *fourth* power of 3.



$3^2 = 3 \times 3 = 9$

798. The ***Square*** of a number is its *second* power, so called because the number of superficial units in a square is equal to the second power of the number of linear units in one of its *sides.*



$3^3 = 3 \times 3 \times 3 = 27$

799. The ***Cube*** of a number is its *third* power, so called because the number of units of volume in a cube is equal to the third power of the number of linear units in one of its *edges.*

800. A ***Perfect Power*** is a number which can be resolved into equal factors. Thus, 25 is a perfect power of the *second* degrée, and 27 is a perfect power of the *third* degree.

801. PRINCIPLE.—*The sum of the exponents of two powers of the same number is equal to the exponent of the product of those powers.* Thus, $2^2 \times 2^3 = 2^5$; for $2^2 = 2 \times 2$, and $2^3 = 2 \times 2 \times 2$; hence $2^2 \times 2^3 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5$.

WRITTEN EXERCISES.

802. To find any power of a number.

1. Find the third power of 35.

OPERATION.

$35 = 35^1$; $35 \times 35 = 35^2 = 1225$

$1225 \times 35 = 35^3 = 42875$

ANALYSIS.—Since using any number three times as a factor produces the third power of that number (**797**), $35 \times 35 \times 35 = 35^3 = 42875$.

2. Find the square of 37. Of 42. Of 56. Of 75.

3. Find the cube of 15. Of 18. Of 42. Of 54.

4. What is the value of 63^2? of 48^3? of 32^4? of 12^5?

RULE.—*Find the product of as many factors, each equal to the given number, as there are units in the exponent of the required power.*

5. What is the third power of $\frac{4}{5}$?

OPERATION.—$(\frac{4}{5})^3 = \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = \frac{4 \times 4 \times 4}{5 \times 5 \times 5} = \frac{4^3}{5^3} = \frac{64}{125}$.

RULE.—*A fraction may be raised to any power by involving each of its terms separately to the required power.*

6. What is the square of $\frac{7}{16}$? The cube of $\frac{12}{31}$?

7. Raise $\frac{9}{14}$ to the 4th power. $2\frac{1}{2}$ to the 5th power.

Find the required power of the following :

8. 25.4^2.
9. 106^3.
10. $(44\frac{1}{4})^2$.
11. $(\frac{11}{15})^4$.
12. $.0342^2$.
13. $.5^6$.
14. 36.02^3.
15. $.40316^3$
16. $(182\frac{1}{5})^2$.
17. $(4.07\frac{1}{2})^2$.
18. $(1\frac{9}{10})^5$.
19. $.0063^3$.

Find the value of each of the following expressions :

20. $4.6^3 \times 25^3$.

21. $6.75^4 - (7\frac{1}{4})^2$.

22. $\frac{7}{8}$ of $(\frac{4}{5})^3 \times (3\frac{4}{8})^2$.

23. $8^6 \div .4096$.

24. $2.5^3 \times (12\frac{3}{4})^2$.

25. $(7.5)^3 \div (1\frac{1}{2})^3$.

26. $(4^3 \times 5^6 \times 12^3) \div (4^2 \times 10^4 \times 3^2)$.

FORMATION OF SQUARES AND CUBES BY THE ANALYTICAL METHOD.

803. To find the square of a number in terms of its tens and units.

1. Find the square of 27 in terms of its tens and units.

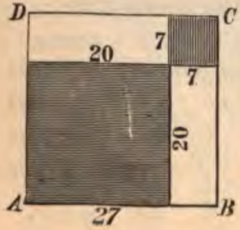
OPERATION.

$$\begin{array}{rr}
27 = & 20+7 \\
27 = & 20+7 \\
\hline
189 = & (20\times 7)+7^2 \\
540 = & 20^2+(20\times 7) \quad \\
\hline
729 = & 20^2+(2\times 20\times 7)+7^2
\end{array}$$

ANALYSIS.—The product of $20+7$ by 7 is $20 \times 7 + 7^2$, and the product of $20+7$ by 20 is $20^2+(20 \times 7)$; hence $20^2+(2\times 20\times 7)+7^2$, which is the sum of these partial products, is the square of $20+7$ or 27.

PRINCIPLE.—*The square of a number consisting of tens and units is equal to the sum of the squares of the tens and units increased by twice their product.*

GEOMETRICAL ILLUSTRATION.



Let ABCD be a square, each side of which is 27 feet, and let lines be drawn as represented in the figure. It is evident that the square ABCD (27^2) is equal to the sum of two squares, one of which is the square of tens (20^2), the other the square of the units (7^2), together with two rectangles each of whose areas is 20×7.

2. What is the square of 37?

$$\begin{array}{r}
30^2 = 900 \\
2 \times 30 \times 7 = 420 \\
7^2 = 49 \\
\hline
37^2 = 1369
\end{array} \quad (\mathbf{803},\ \text{PRIN.})$$

3. Find the square of 42 in terms of its tens and units.

In like manner find the square

4. Of 48.
5. Of 56.
6. Of 98.
7. Of 125.
8. Of 105.
9. Of 225.
10. Of 197.
11. Of 342.

804. To find the cube of a number in terms of its tens and units.

1. Find the cube of 25 in terms of its tens and units.

OPERATION.

$$\begin{array}{rcr}
25^2 & = & 20^2+(2\times 20\times 5)+5^2 \\
25 & = & 20+5 \\
\hline
25^2\times 5 & = & (20^2\times 5)+(2\times 20\times 5^2)+5^3 \\
25^2\times 20 & = & 20^3+(2\times 20^2\times 5)+\quad(20\times 5^2) \\
\hline
25^3 & = & 20^3+(3\times 20^2\times 5)+(3\times 20\times 5^2)+5^3
\end{array}$$

ANALYSIS.—The *square* of 25 is $20^2+(2\times 20+5)+5^2$. (**803**, PRIN.) Multiplying this by $20+5$ gives the *cube* of 25.

2. Find the cube of 34 in terms of its tens and units.

PRINCIPLE.—*The cube of a number consisting of tens and units is equal to the cube of the tens, plus three times the product of the square of the tens by the units, plus three times the product of the tens by the square of the units, plus the cube of the units.*

GEOMETRICAL ILLUSTRATION.

FIG. 1.

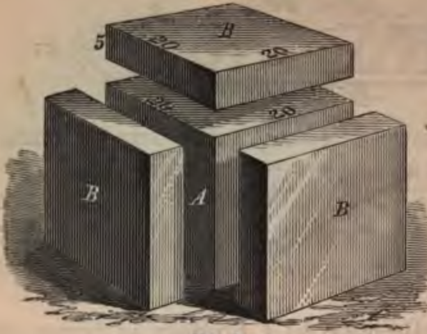


FIG. 2.

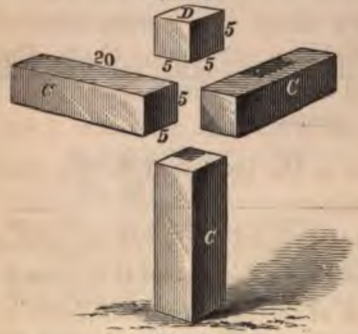
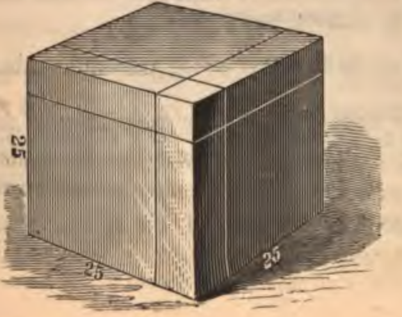


FIG. 3.



The volume of the cube marked A, Fig. 1, is 20^3; the volume of each of the rectangular solids marked B is $20 \times 20 \times 5$, or $20^2 \times 5$; the volume of each of the rectangular solids marked C, in Fig. 2, is $20 \times 5 \times 5$, or 20×5^2; and the volume of the small cube marked D is 5^3. It is evident, that if all these solids are put together as represented in Fig. 3, a cube will be formed, each edge of which is 25.

3. Find the cube of 46?

OPERATION.

$$\begin{array}{r} 40^3 = 64000 \\ 40^2 \times 6 \times 3 = 28800 \\ 40 \times 6^2 \times 3 = 4320 \\ 6^3 = 216 \\ \hline 46^3 = 97336 \end{array}$$

In like manner find the cube

4. Of 48.
5. Of 64.
6. Of 95.
7. Of 125.

EVOLUTION

805. 1. What are the two equal factors of 25? 36?

2. What are the three equal factors of 27? 64? 125?

3. What are the four equal factors of 16? 81? 256?

4. Of what is 81 the 2d power? The 4th power?

DEFINITIONS.

806. The ***Square Root*** of a number is one of the two equal factors of that number; the ***Cube Root*** is one of the three equal factors of that number, etc.

Thus, 3 is the square root of 9, 2 is the cube root of 8, etc.

807. ***Evolution*** is the process of finding the root of any power of a number.

808. The ***Radical Sign*** is $\sqrt{\ }$. When prefixed to a number, it indicates that some root of it is to be found.

809. The ***Index*** of the root is a small figure placed above the radical sign to denote what root is to be found. When no index is written, the index 2 is understood.

Thus, $\sqrt{100}$ denotes the *square root* of 100; $\sqrt[3]{125}$ denotes the *cube root* of 125; $\sqrt[4]{256}$ denotes the *fourth root* of 256; and so on.

Evolution, or both involution and evolution, may be indicated in the same expression by a *fractional exponent*, the numerator denoting the required *power* of the given number, and the denominator the *root* of that power of the number. Thus,

$9^{\frac{1}{2}}$ is equivalent to $\sqrt{9}$; $64^{\frac{1}{3}}$, to $\sqrt[3]{64}$; and $8^{\frac{2}{3}}$, to the cube root *of the second power* of 8, equivalent to $\sqrt[3]{8^2}$, etc.

EVOLUTION BY FACTORING.

WRITTEN EXERCISES.

810. To find any root of a number by factoring.

1. Find the cube root of 1728.

OPERATION.

3)1728
3)576
3)192
2)64
2)32
2)16
2)8
2)4
2

ANALYSIS.—A number that is a perfect cube, is composed of *three* equal factors, and one of them is the cube root of that number.

The prime factors of 1728 are 3, 3, 3, 2, 2, 2, 2, 2, 2; hence 1728 = (3 × 2 × 2) × (3 × 2 × 2) × (3 × 2 × 2); therefore the cube root of 1728 is (3 × 2 × 2), or 12.

RULE.—*Resolve the given number into its prime factors; then, to produce the square root, take one of every two equal factors; to produce the cube root take one of every three equal factors; and so on.*

2. Find the square root of 64. Of 256. Of 576. Of 6561.

3. Find the cube root of 729. Of 2744. Of 9261. Of 3375.

GENERAL METHOD OF SQUARE ROOT.

811. A ***Perfect Square*** is a number which has an exact square root.

812. PRINCIPLES.—1. *The square of a number expressed by a single figure contains no figure of a higher order than tens.*

2. *The square of tens contains no significant figure of a lower order than hundreds, nor of a higher order than thousands.*

3. *The square of a number contains twice as many figures as the number, or twice as many less one.* Thus,

$1^2 = 1$, $10^2 = 100$,
$9^2 = 81$, $100^2 = 10000$,
$99^2 = 9801$, $1000^2 = 1000000$.

Hence,

4. *If any perfect square be separated into periods of two figures each, beginning with units' place, the number of periods will be equal to the number of figures in the square root of that number.*

If the number of figures in the number is *odd*, the left-hand period will contain only one figure.

WRITTEN EXERCISES.

813. To find the square root of a number.

1. Find the square root of 4356.

OPERATION.

43,56 (60 + 6 = 66
$60^2 =$ 3600
120 + 6 = 126) 756
756

ANALYSIS.—Since 4356 consists of two periods, its square root will consist of two figures (**812,** PRIN. 4). Since 56 cannot be a part of the square of the tens (**812,** PRIN. 2), the tens of the root must be found from the first period 43.

The greatest number of tens whose square is contained in 4300 is 6. Subtracting 3600, which is the square of 6 tens, from the given number, the remainder is 756. This remainder is composed of *twice* the product of the tens by the units, and the square of the units (**803,** PRIN.). But the product of tens by units cannot be of a lower order than tens; hence the last figure 6 cannot be a part of twice the product of the tens by the units; this double product must therefore be found in the part 750.

Now, if we double the tens of the root and divide 750 by the result, *the quotient 6* will be the units' figure of the root, or a figure greater than the units' figure. This quotient cannot be too small, for the part 750 is at least equal to twice the product of the tens by the units; but it may be too large, for the part 750, besides the double product of the tens by the units, may contain tens arising from the square of the units. (**812,** PRIN. 1.) Subtracting $6\times120+6^2$ or $6\times\overline{120+6}$ from 756, nothing remains. Hence 66 is the required root.

1. In this example, 120 is a *partial* or *trial divisor*, and 126 is a *complete divisor*.

2. If the root contains more than two figures, it may be found by a similar process, as in the following example, where it will be seen that the partial divisor at each step is obtained by doubling that part of the root already found.

2. Find the square root of 186624.

OPERATION.

$$\begin{array}{rl}
 & 18,66,24\,(400+30+2=432 \\
 & \underline{16\;00\;00} \\
400\times2+30=830) & 2\;66\;24 \\
 & \underline{2\;49\;00} \\
400\times2+30\times2+2=862) & 1724 \\
 & \underline{1724}
\end{array}$$

The ciphers on the right are usually omitted for the sake of brevity. Thus,

$$\begin{array}{rl}
 & 18,66,24\,(432 \\
 & \underline{16} \\
83) & 266, \text{ etc.}
\end{array}$$

3. Find the square root of 7225.

4. What is the square root of 58564.

RULE.—I. *Separate the given number into periods of two figures each, beginning at the units' place.*

II. *Find the greatest number whose square is contained in the period on the left; this will be the first figure in the root. Subtract the square of this figure from the period on the left, and to the remainder annex the next period to form a dividend.*

III. *Divide this dividend, omitting the figure on the right, by double the part of the root already found, and annex the quotient to that part, and also to the divisor; then multiply the divisor thus completed by the figure of the root last obtained, and subtract the product from the dividend.*

IV. *If there are more periods to be brought down, continue the operation in the same manner as before.*

1. If a cipher occur in the root, annex a cipher to the trial divisor, and another period to the dividend, and proceed as before.

2. If there is a remainder after the root of the last period is found, annex periods of ciphers and continue the root to as many decimal places as are required.

Find the square root

5. Of 9604.
6. Of 13225.
7. Of 11881.
8. Of 994009.
9. Of 2050624.
10. Of 29855296.

11. Find the square root of $\frac{100}{121}$.

Operation.—$\sqrt{\frac{110}{121}} = \frac{\sqrt{100}}{\sqrt{121}} = \frac{10}{11}$.

Rule.—*The square root of a fraction may be found by extracting the square root of the numerator and denominator separately.*

Mixed numbers may be reduced to the decimal form before extracting the root; or, if the denominator of the fraction is a perfect square, to an improper fraction.

In extracting the square root of a number containing a *decimal*, begin at the units' place, and proceed both toward the left and the right to separate into periods, then proceed as in the extraction of the square root of integers.

Extract the square root

12. Of $\frac{169}{196}$.
13. Of $\frac{625}{6561}$.
14. Of $\frac{576}{18225}$.
15. Of .001225.
16. Of 196.1369.
17. Of 2.251521.
18. Of 58.1406¼.
19. Of 17¾.
20. Of 10795.21.

21. What is the square root of 3486784401?

22. What is the square root of 9.0000994009?

23. Find the value of $32^{\frac{1}{2}}$ to 6 decimal places.

24. Find the square root of $2\frac{3}{8}$ to 4 decimal places.

25. Find the square root of $\frac{5}{6}$ to 5 decimal places.

26. Find the value of $.125^{\frac{2}{3}}$ to 5 decimal places.

Find the second member of the following equations:

27. $\sqrt{.1369} + \sqrt{1296} = ?$

28. $(36\frac{9}{25})^{\frac{1}{2}} \times \sqrt{.25^2} = ?$

29. $2.8^3 \div \sqrt{.117649} = ?$

30. $\sqrt{\frac{3136}{7056}} - (\frac{225}{2025})^{\frac{1}{2}} \div 2^2 = ?$

31. $\frac{9^{\frac{1}{2}}}{\sqrt{3^2}} \times \frac{3^2}{\sqrt{9^2}} = ?$

32. $\sqrt{2.6896} + .3729 \times \frac{3}{4}$ of $\sqrt{.256} = ?$

33. $(7.2 - \sqrt{27.04})^5 \div (\frac{2}{3})^2 = ?$

34. $(\sqrt{81} - 16^{\frac{1}{2}}) \times (\sqrt{169} + 25^{\frac{1}{2}}) = ?$

35. $\sqrt{264^2} \times 4.41 \div (5.3361)^{\frac{1}{2}} - (2.3^3 \times \sqrt{3.61}) = ?$

GEOMETRICAL EXPLANATION OF SQUARE ROOT.

814. What is the length of one side of a square whose area is 729 square feet?

FIG. 1.

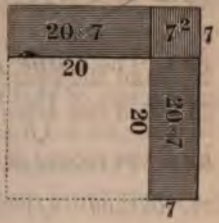
Let Fig. 1 represent a square whose area is 729 square feet. It is required to find the length of one side of this square.

Since the area of a square is equal to the square of one of its sides, a side may be found by extracting the square root of the area.

Since 729 consists of two periods, its square root will consist of two figures. The greatest number of *tens* whose square is contained in 700 is 2. Hence the length of the side of the square is *20 feet plus the* units' figure of the root.

Removing the square whose side is 20 feet and whose area is 400 square feet, there remains a surface whose area is 329 square feet (Fig. 2). This remainder consists of two equal rectangles, each of which is 20 feet long, and a square whose side is equal to the width of each rectangle. The units' figure of the root is equal to the width of one of these rectangles.

FIG. 2.



The area of a rectangle is equal to the product of its length and width (**462**); hence, if the area be divided by the length, the quotient will be the width. Now, since the two rectangles contain the greater portion of the 329 square feet, 2×20 or 40, the length of the two rectangles, may be used as a trial divisor to find the width. Dividing 329 by 40, the quotient is 8. But this quotient is too large for the width of the rectangles, for if 8 feet is the width, the area of Fig. 2 will be $40\times8+8^2$ or 384 square feet. Taking 7 feet for the width of the rectangles, the area of Fig. 2 is $40\times7+7^2$ or 329 square feet. Hence $20+7$ or 27 feet is the length of a side of the square whose area is 729 square feet.

PROBLEMS.

815. 1. A square field contains 1016064 square feet. What is the length of each side?

2. A square farm contains 361 A. Find the length of one side.

3. A field is 208 rd. long and 13 rd. wide. What is the length of the side of a square containing an equal area?

4. If 251 A. 65 P. of land are laid out in the form of a square, what will be the length of each of its sides?

5. A circular island contains 21170.25 P. of land. What is the length of the side of a square field of equal area?

6. If it cost $312 to enclose a field 216 rd. long and 24 rd. wide, what will it cost to enclose a square field of equal area with the same kind of fence?

CUBE ROOT.

816. A ***Perfect Cube*** is a number which has an exact cube root.

817. The ***Cube Root*** of a number is one of the *three* equal factors of that number. Thus, the cube root of 125 is 5, since $5 \times 5 \times 5 = 125$.

818. PRINCIPLES.—1. *The cube of a number expressed by a single figure contains no figure of a higher order than hundreds.*

2. *The cube of tens contains no significant figure of a lower order than thousands, nor of a higher order than hundred thousands.*

3. *The cube of a number contains three times as many figures as the number, or three times as many, less one or two.* Thus,

$1^3 = 1$ $10^3 = 1,000$
$3^3 = 27$ $100^3 = 1,000,000$
$9^3 = 729$ $1000^3 = 1,000,000,000$
$99^3 = 907,299$ $10000^3 = 1,000,000,000,000$

4. *If any perfect cube be separated into periods of three figures each, beginning with units' place, the number of periods will be equal to the number of figures in the cube root of that number.*

WRITTEN EXERCISES.

819. To find the cube root of a number.

1. Find the cube root of 405224.

OPERATION.

$$\begin{array}{rl} & 405{,}224\ (\ 70 + 4 = 74,\text{ cube root.} \\ 70^3 = & 343\ 000 \\ \hline 70^2 \times 3 = 14700\ ) & 62\ 224 \\ \hline 74^3 = & 405\ 224 \end{array}$$

Analysis.—Since 405224 consists of two periods, its cube root will consist of two figures (**818**, Prin. 4). Since 224 cannot be a part of the cube of the tens of the root (**818**, Prin. 2), the first figure of the root must be found from the first period, 405. The greatest number of *tens* whose cube is contained in 405000 is 7. Subtracting the cube of 7 tens from the given number, the remainder is 62224. This remainder is equal to the product of three times the square of the tens of the root by the units, plus three times the product of the tens by the square of the units, plus the cube of the units (**804**, Prin.) But the product of the square of tens by units cannot be of a lower order than hundreds (**818**, Prin. 2); hence the number represented by the last two figures, 24, cannot be a part of three times the product of the square of the tens of the root by the units; the triple product must therefore be found in the part 62200. Hence, if 62200 be divided by 3×70^2, the quotient, which is 4, will be the units' figure of the root or a figure greater than the units' figure. Subtracting 74^3 from the given number, the result is 0; hence 74 is the required root.

Instead of cubing 74, the parts which make up the remainder 62224 may be formed and added thus:

$$\begin{array}{rr} 3 \times 70^2 \times 4 = & 58800 \\ 3 \times 70 \times 4^2 = & 3360 \\ 4^3 = & \underline{64} \\ & 62224; \end{array}$$

Or, since 4 is a common factor in the three parts which make up the remainder, these parts may be combined thus:

$$\begin{array}{rl} 3 \times 70^2 \quad = & 14700 \\ 3 \times 70 \times 4 = & 840 \\ 4^2 = & \underline{16} \\ & 15556 \times 4 = 62224. \end{array}$$

1. In this example, 14700 is a *partial* or *trial divisor*, and 15556 is a *complete divisor*.

2. If the cube root contains more than two figures, it may be found by a similar process, as in the following example, where it will be seen that the partial divisor at each step is equal to *three times the square of that* part of the root already found.

2. Find the cube root of 12812904.

OPERATION.

			Cube Root.
			12,812,904 ($200+30+4=234$
	$200^3=$		8 000 000
1ST PAR. DIVISOR	3×200^2	$=120000$	4 812 904
	$3\times200\times30=$	18000	
	$30^2=$	900	4167000
1ST COMPLETE DIVISOR		138900	645904
2D PAR. DIVISOR	3×230^2	$=158700$	
	$3\times230\times4=$	2760	
	$4^2=$	16	645904
2D COMPLETE DIVISOR		161476	

The operation may be abridged as follows:

			12,812,904 (234
	$2^3=$		8
1ST PARTIAL DIVISOR	3×20^2	$=1200$	4812
	$3\times20\times3=$	180	
	$3^2=$	9	4167
1ST COMPLETE DIVISOR		1389	645904
2D PAR. DIVISOR	3×230^2	$=158700$	
	$3\times230\times4=$	2760	
	$4^2=$	16	645904
2D COMPLETE DIVISOR		161476	

RULE.—I. *Separate the given number into periods of three figures each, beginning at the units' place.*

II. *Find the greatest number whose cube is contained in the period on the left; this will be the first figure in the root. Subtract the cube of this figure from the period on the left, and to the remainder annex the next period to form a dividend.*

III. *Divide this dividend by the partial divisor, which is 3 times the square of the root already found, considered as tens; the quotient is the* second figure of the root.

IV. *To the partial divisor add* 3 *times the product of the second figure of the root by the first considered as tens, also the square of the second figure, the result will be the complete divisor.*

V. *Multiply the complete divisor by the second figure of the root and subtract the product from the dividend.*

VI. *If there are more periods to be brought down, proceed as before, using the part of the root already found, the same as the first figure in the previous process.*

1. If a cipher occur in the root, annex two ciphers to the trial divisor, and another period to the dividend; then proceed as before, annexing both cipher and trial figure to the root.

2. If there is a remainder after the root of the last period is found, annex periods of ciphers and proceed as before. The figures of the root thus obtained will be *decimals.*

What is the cube root

3. Of 15625?
4. Of 166375?
5. Of 1030301?
6. Of 4492125?
7. Of 1045678375?
8. Of 4080659192?

9. Find the cube root of $\frac{8}{27}$.

$$\text{Operation.}—\sqrt[3]{\tfrac{8}{27}} = \frac{\sqrt[3]{8}}{\sqrt[3]{27}} = \tfrac{2}{3}.$$

Rule.—*The cube root of a fraction may be found by extracting the cube root of the numerator and denominator.*

In extracting the cube root of *decimal* numbers, begin at the units' place and proceed both toward the left and the right, to separate into periods of *three* figures each.

Extract the cube root

10. Of $\frac{1000}{1331}$.
11. Of $\frac{13824}{15625}$.
12. Of $2\frac{7}{8}$.
13. Of 39304.
14. Of .091125.
15. Of 12.812904.

16. What is the cube root of 98867482624?

17. What is the cube root of .000529475129?

18. *Find the* cube root of $\frac{1}{2}$ correct to 4 decimal places.

Find the second member of the following equations:

19. $1.44^{\frac{1}{3}} + 2.5^{\frac{5}{3}} = ?$

20. $\sqrt[3]{\frac{1331}{3375}} \times \sqrt[3]{\frac{125}{512}} = ?$

21. $\sqrt[3]{.4096 - .2368} = ?$

22. $\sqrt[3]{54.872} - (21.952)^{\frac{1}{3}} = ?$

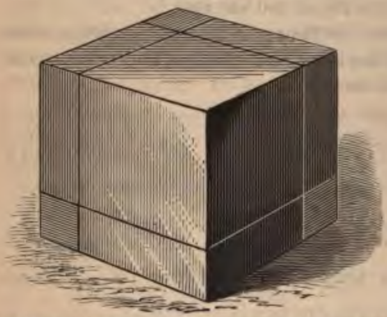
23. $24.8 + \sqrt[3]{103.823} \times (.125)^{\frac{1}{3}} = ?$

24. $\sqrt[3]{16^6} \div \sqrt[3]{64} - 4 \times \sqrt[3]{.512} = ?$

GEOMETRICAL EXPLANATION OF CUBE ROOT.

820. What is the length of the edge of a cube whose volume is 15625 cubic feet?

FIG. 1.



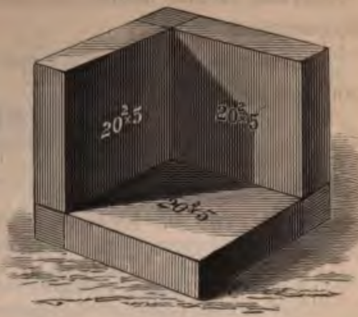
Let Fig. 1 represent a cube whose volume is 15625 cubic feet. It is required to find the length of the edge of this cube.

Since the volume of a cube is equal to the cube of one of its edges, an edge may be found by extracting the cube root of the volume.

Since 15625 consists of two periods, its cube root will consist of two figures. The greatest number of *tens* whose cube is contained in 15000 is 2. Hence, the length of the edge of the cube is 20 feet plus the units' figure of the root. Removing the cube whose edge is 20 feet and whose volume is 8000 cubic feet, there remains a solid whose volume is 7625 cubic feet (Fig. 2). This remainder consists of solids similar to those marked B, C, and D, in Fig. 1 and Fig. 2 of Art. **804.**

$$\begin{array}{rr|l}
 & & 15{,}625\,(25 \\
 & 2^3 = & 8 \\ \hline
3 \times 20^2 & = 1200 & 7625 \\
3 \times 20 \times 5 & = 300 & \\
5^2 & = 25 & \\ \cline{2-2}
 & 1525 & 7625 \\ \cline{3-3}
\end{array}$$

FIG. 2.



The volume of a rectangular solid is equal to the product of the area of its base by its height or thickness (**472**); hence, if the volume be divided by the area of the base the quotient will be the thickness. Now, since the three equal rectangular solids, each of which is 20 feet square and whose thickness is the units' figure of the root, contain the greater portion of the 7625 cubic feet, 3×20^2 or 300×2^2 may be used as a trial divisor to find the thickness. Dividing 7625 by 1200 the quotient is 6. But this quotient is too large, for if 6 feet is the thickness, the volume of Fig. 2 will be $3 \times 20^2 \times 6 + 3 \times 20 \times 6^2 + 6^3$, or 9576 cubic feet. Taking 5 feet for the thickness, the volume of Fig. 2 is 7625 cubic feet, for $3 \times 20^2 \times 5 + 3 \times 20 \times 5^2 + 5^3 = (300 \times 2^2 + 30 \times 2 \times 5 + 5^2)\,5 = 1525 \times 5 = 7625$. Hence, 25 feet is the length of the edge of a cube whose volume is 15625 cubic feet.

PROBLEMS.

821. 1. What is the length of the edge of a cubical box that contains 46656 cu. inches?

2. What must be the length of the edge of a cubical bin that shall contain the same volume as one that is 16 ft. long, 8 ft. wide, and 4 ft. deep?

3. What are the dimensions of a cube that has the same volume as a box 2 ft. 8 in. long, 2 ft. 3 in. wide, and 1 ft. 4 in. deep?

4. How many square feet in the surface of a cube whose volume is 91125 cubic feet?

5. What is the length of the inner edge of a cubical bin that *contains* 150 bushels?

6. What is the depth of a cubical cistern that holds 200 barrels of water?

7. Find the length of a cubical vessel that will hold 4000 gallons of water.

ROOTS OF HIGHER DEGREE.

822. Any root whose index contains no other factors than 2 or 3 may be extracted by means of the square and cube roots.

If any power of a given number is raised to any required power, the result is that power of the given number denoted by the *product* of the two exponents. (**801.**) Conversely, if two or more roots of a given number are extracted, successively, the result is that root of the given number denoted by the product of the indices.

1. What is the 6th root of 2176782336?

OPERATION.

$$\sqrt[2]{2176782336} = 46656$$
$$\sqrt[3]{46656} = 36$$

Or,

$$\sqrt[3]{2176782336} = 1296$$
$$\sqrt[2]{1296} = 36$$

ANALYSIS.—The index of the required root is 6 = 2 × 3; hence extract the square root of the given number, and the cube root of this result, which gives 36 as the 6th or required root. Or, first find the cube root of the given number, and then the square root of the result.

RULE.—*Separate the index of the required root into its prime factors, and extract successively the roots indicated by the several factors obtained; the final result will be the required root.*

2. What is the 4th root of 5636405776?

3. What is the 8th root of 1099511627776?

4. What is the 6th root of 25632972850442049?

5. What is the 9th root of 1.577635?

For further practical applications of Involution and Evolution, see "Mensuration."

PROGRESSIONS

823. An ***Arithmetical Progression*** is a succession of numbers, each of which is greater or less than the preceding one by a constant *difference.*

Thus, 5, 7, 9, 11, 13, 15, is an arithmetical progression.

824. The ***Terms*** of an arithmetical progression are the numbers of which it consists. The first and last terms are called the *Extremes,* and the other terms the *Means.*

825. The ***Common Difference*** is the difference between any two consecutive terms of the progression.

826. An ***Increasing Arithmetical Progression*** is one in which each term is greater than the preceding one.

Thus, 1, 3, 5, 7, 9, 11, is an increasing progression.

827. A ***Decreasing Arithmetical Progression*** is one in which each term is less than the preceding one.

Thus, 15, 13, 11, 9, 7, 5, 3, 1, is a decreasing progression.

828. The following are the quantities considered in arithmetical progression :

1. The first term (a).
2. The last term (l).
3. The common difference (d).
4. The number of terms (n).
5. The sum of all the terms (s).

WRITTEN EXERCISES.

829. To find one of the extremes, when the other extreme, the common difference, and the number of terms are given.

1. The first term of an increasing progression is 8, the common difference 5, and the number of terms 20; what is the last term?

OPERATION.

$$20 - 1 = 19$$

$$\overline{19 \times 5} + 8 = 103 = l.$$

ANALYSIS.—The 2d term is 8+5; the 3d term is 8+(5×2) the 4th term is 8+(5×3); and so on. Hence 8+(19×5) or 103 is the 20th or *last term.*

2. The last term of an increasing progression is 103, the common difference 5, and the number of terms 20; what is the first term?

OPERATION.

$$20 - 1 = 19$$

$$103 - \overline{19 \times 5} = 8 = a$$

ANALYSIS.—The 1st term must be a number to which, if 19×5 be added, the sum shall be 103; hence, if 19×5 is subtracted from 103, the remainder is the *first term.*

3. The first term of a decreasing progression is 203, the common difference 5, and the number of terms 40; what is the last term?

4. The last term of a decreasing progression is 1, the common difference 2, and the number of terms 9; what is the first term?

RULE.—I. *If the given extreme is the less, add to it the product of the common difference by the number of terms less one.*

II. *If the given extreme is the greater, subtract from it the product of the common difference by the number of terms less one.*

FORMULÆ.— $\begin{cases} l = a + (n-1) \times d. \\ a = l - (n-1) \times d. \end{cases}$

5. The first term of an increasing progression is 5, the common difference 4, and the number of terms 8; what is the last term?

6. The first term of an increasing progression is 2, and the common difference 3; what is the 50th term?

7. The first term of a decreasing progression is 100, and the common difference 7; what is the 13th term?

8. The first term of an increasing progression is $\frac{2}{3}$, the common difference $\frac{3}{8}$, and the number of terms 20; what is the last term?

830. To find the common difference, when the extremes and number of terms are given.

1. The extremes of a progression are 8 and 103, and the number of terms 20; what is the common difference?

OPERATION.

$$\overline{103 - 8} \div 19 = 5 = d$$

ANALYSIS.—The difference between the extremes is equal to the product of the common difference by the number of terms less one (**829**); hence the *common difference* is $\frac{95}{19}$, or 5.

2. The extremes of a progression are 1 and 17, and the number of terms 9; what is the common difference?

RULE.—*Divide the difference between the extremes by the number of terms less one.*

FORMULA.—$d = \frac{l - a}{n - 1}$.

3. The extremes are 3 and 15, and the number of terms 7; what is the common difference?

4. The extremes are 1 and 51, and the number of terms 76; what is the common difference?

5. The youngest of ten children is 8, and the eldest 44 years old; their ages are in arithmetical progression. What is the common difference of their ages?

6. The amount of $800 for 60 years, at simple interest, is $4160. What is the rate per cent.?

7. The extremes are 0 and $2\frac{1}{2}$, and the number of terms 18; what is the common difference?

831. To find the number of terms, when the extremes and common difference are given.

1. The extremes of a progression are 8 and 103, and the common difference 5; what is the number of terms?

OPERATION.

$$\overline{103-8}\div 5=19$$
$$19+1=20=n$$

ANALYSIS.—The difference between the extremes is equal to the product of the common difference by the number of terms less one (**830**); hence the number of terms less one is equal to $\frac{95}{5}$ or 19; therefore 19 + 1 or 20 is the *number of terms*.

2. The extremes of a progression are 1 and 17, and the common difference 2; what is the number of terms?

RULE.—*Divide the difference between the extremes by the common difference, and add one to the quotient.*

FORMULA.—$n=\dfrac{l-a}{d}+1.$

3. The extremes are 5 and 75, and the common difference is 5; what is the number of terms?

4. The extremes are $\frac{1}{2}$ and 20, and the common difference is $6\frac{1}{2}$; what is the number of terms?

5. A laborer received 50 cents the first day, 54 cents the second, 58 cents the third, and so on, until his wages were $1.54 a day; how many days did he work?

6. In what time will $500, at 7 per cent. simple interest, amount to $885?

832. **To find the sum of all the terms, when the extremes and the number of terms are given.**

1. The extremes of an arithmetical progression are 2 and 14, and the number of terms is 5; what is the sum of all the terms?

OPERATION.

$$\begin{aligned} s &= 2+5+8+11+14 \\ s &= \underline{14+11+8+5+2} \\ 2\,s &= 16+16+16+16+16 \\ 2\,s &= 16\times5=(2+14)\times5 \\ s &= \frac{2+14}{2}\times5=40. \end{aligned}$$

ANALYSIS.—The common difference is found to be 3 (**830**); hence the required sum is equal to $2+5+8+11+14$, or $14+11+8+5+2$. Adding the corresponding terms of these two progressions, we have 2 times the sum $=16\times5=(2+14)\times5$; hence the sum is $\frac{2+14}{2}\times5=40$.

2. The extremes of an arithmetical progression are 5 and 75, and the number of terms is 15; what is the sum of all the terms?

RULE.—*Multiply the sum of the extremes by half the number of terms.*

FORMULA.—$s=\frac{n}{2}\times(a+l)$.

3. The extremes are 4 and 40, and the number of terms is 7; what is the sum of all the terms?

4. The extremes are 0 and 250, and the number of terms is 1000; what is the sum of all the terms?

5. How many strokes, beginning at 1 o'clock, does the hammer of a common clock strike in 12 hours?

6. A body will fall $16\frac{1}{12}$ ft. in the first second of its fall, $48\frac{1}{4}$ ft. in the second second, $80\frac{5}{12}$ ft. in the third second, and so on; how far will it fall in one minute?

833. A ***Geometrical Progression*** is a succession of numbers, each of which is greater or less than the preceding one in a constant *ratio.*

Thus, 1, 3, 9,27, 81, etc., is a geometrical progression.

834. The ***Terms*** of a geometrical progression are the numbers of which the progression consists. The first and last terms are called the *Extremes,* and the other terms the *Means.*

835. The ***Ratio*** of a geometrical progression is the quotient obtained by dividing any term by the preceding one.

836. An ***Increasing Geometrical Progression*** is one in which the ratio is greater than 1.

Thus, 1, 2, 4, 8, 16, etc., is an increasing progression.

837. A ***Decreasing Geometrical Progression*** is one in which the ratio is less than 1.

Thus, 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, etc., is a decreasing progression.

838. An ***Infinite Decreasing Geometrical Progression*** is one in which the ratio is less than 1, and the number of terms *infinite.*

Thus, 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$, and so on is an infinite decreasing progression.

839. The following are the quantities considered in geometrical progression :

1. The first term (a).
2. The last term (l).
3. The ratio (r).
4. The number of terms (n).
5. The sum of all the terms (s).

WRITTEN EXERCISES.

340. To find one of the extremes, when the other extreme, the ratio, and the number of terms are given.

1. The first term of a progression is 2, the ratio 3, and the number of terms 10; what is the last term?

OPERATION.

$3^9 = 19683$

$\underline{\quad\quad\quad 2}$

$39366 = l$

ANALYSIS.—The 2d term is 2×3; the third term is $2 \times 3 \times 3$ or 2×3^2; the 4th term is 2×3^3; and so on. Hence the 10th or *last term* is 2×3^9 or 39366.

2. The last term of a progression is 39366, the ratio 3, and the number of terms 10; what is the first term?

OPERATION.

$\frac{39366}{3^9} = 2 = a$

ANALYSIS.—The first term must be a number, by which if 3^9 be multiplied the product shall be 39366; hence, if 39366 be divided by 3^9, the quotient will be the *first term*.

3. The first term of a progression is 1, the ratio $\frac{1}{2}$, and the number of terms 9; what is the last term?

RULE.—I. *If the given extreme is the first term, multiply it by that power of the ratio whose exponent is one less than the number of terms.*

II. *If the given extreme is the last term, divide it by that power of the ratio whose exponent is one less than the number of terms.*

FORMULÆ.—$l = ar^{n-1}$; $\quad a = \frac{l}{r^{n-1}}$.

4. The first term of a geometrical progression is 6, the ratio 4, the number of terms 6; what is the last term?

5. The last term is 192, the ratio 2, and the number of terms 7; what is the first term?

6. A drover bought 20 cows, agreeing to pay $1 for the first, $2 for the second, $4 for the third, and so on ; how much did he pay for the last cow ?

7. Find the amount of $250 for 4 years at 6 per cent. compound interest.

The first term is 250, the ratio 1.06, and the number of terms 5.

8. If 1 cent had been put at interest in 1634, what would it have amounted to in the year 1874, if it had doubled its value every 12 years ?

841. To find the ratio, when the extremes and the number of terms are given.

1. The first term is 2, the last term 512, and the number of terms 5 ; what is the ratio ?

OPERATION.

$$\tfrac{512}{2} = 256$$

$$\sqrt[4]{256} = 4 = r$$

ANALYSIS.—If the 4th power of the ratio be multiplied by 2, the product is 512 (**840**) ; hence, if 512 be divided by 2, the quotient, 256, is the 4th power of the ratio. Hence the *ratio* is the 4th root of 256, or 4.

2. The first term is 1, the last term $\frac{1}{256}$, and the number of terms 9 ; what is the ratio ?

RULE.—*Divide the last term by the first, and extract that root of the quotient whose index is one less than the number of terms.*

FORMULA.—$r = \sqrt[n-1]{\frac{l}{a}}$.

3. The first term is 8, the last term 5000, and the number of terms 5 ; what is the ratio ?

4. The first term is .0112, the last term 7, and the number of terms 5 ; what is the ratio ?

5. The first term is $\frac{1}{48}$, the last term $15\frac{3}{16}$, and the number of terms 7 ; what is the ratio ?

842. **To find the number of terms, when the extremes and the ratio are given.**

1. The extremes are 2 and 512, and the ratio is 4; what is the number of terms?

OPERATION.

$$\begin{array}{r} 2\,)\,\underline{512} \\ 256 \\ 4^4 = 256 \end{array}$$

ANALYSIS.—If 512 be divided by 2, the quotient, 256, is that power of the ratio whose exponent is one less than the number of terms (**841**). But 256 is the 4th power of the ratio 4; hence the *number of terms* is 5.

2. The extremes are 1 and $\frac{1}{256}$, and the ratio is $\frac{1}{2}$; what is the number of terms?

RULE.—*Divide the last term by the first; then the exponent of the power to which the ratio must be raised to produce the quotient is one less than the number of terms.*

FORMULA.—$r^{n-1} = \frac{l}{a}$.

3. The extremes are 2 and 1458, and the ratio is 3; what is the number of terms?

4. The extremes are $\frac{1}{640}$ and $\frac{1}{5}$, and the ratio 2; what is the number of terms?

843. **To find the sum of all the terms, when the extremes and the ratio are given.**

1. The extremes are 2 and 128, and the ratio is 4; what is the sum of all the terms?

OPERATION.

$$\frac{(128 \times 4) - 2}{4 - 1} = \frac{510}{3} = 170 = s$$

$$\begin{aligned} 4\,s &= \phantom{2+{}} 8+32+128+512 \\ s &= 2+8+32+128 \\ \hline 3\,s &= 512 - 2 = 510 \\ \tfrac{510}{3} &= 170 = s \end{aligned}$$

ANALYSIS.—Subtract the sum from 4 times the sum, and 510 remains, which is 3 times the sum; hence $\frac{510}{3}$ or 170 is the *sum*.

2. The extremes are 1 and $\frac{1}{16}$, and the ratio is $\frac{1}{2}$; what is the sum of all the terms?

$$\begin{array}{rl} s = & 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \\ \frac{1}{2}\,s = & \underline{\quad\;\; \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32}} \\ \frac{1}{2}\,s = & 1 - \frac{1}{32} = \frac{31}{32};\ \frac{31}{32} \div \frac{1}{2} = 1\frac{15}{16}. \end{array}$$

RULE.—*Multiply the last term by the ratio, and divide the difference between the product and the first term by the difference between* 1 *and the ratio.*

FORMULA.—$s = \dfrac{lr - a}{r - 1}$.

3. The extremes are 3 and 384, and the ratio is 2; what is the sum of all the terms?

4. The extremes are $4\frac{4}{5}$ and $\frac{8}{405}$, and the ratio is $\frac{1}{3}$; what is the sum of all the terms?

5. What is the sum of all the terms of the infinite progression 8, 4, 2, 1, $\frac{1}{2}$, $\frac{1}{4}$, ... ?

The *last term* of this progression may be conceived as 0.

6. What is the sum of all the terms of the infinite progression 1, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, $\frac{1}{81}$, ... ?

7. What is the sum of $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8}$, etc., to infinity?

8. The first is 7, the ratio 3, and the number of terms 4; what is the sum of all the terms?

First find the last term by Art. **340.**

9. A drover bought 10 cows, agreeing to pay $1 for the first, $2 for the second, $4 for the third, and so on; what did he pay for the 10 cows?

10. If a man were to buy 12 horses, paying 2 cents for the first horse, 6 cents for the second, and so on, what would they cost him?

ANNUITIES

844. An ***Annuity*** is a sum of money payable annually. The term is also applied to a sum of money payable at any equal intervals of time.

845. A ***Certain Annuity*** is one which continues for a definite period of time.

846. A ***Perpetual Annuity*** or ***Perpetuity*** is one which continues forever.

847. A ***Contingent Annuity*** is one which begins or ends, or both begins and ends, on the occurrence of some specified future event or events.

848. An ***Annuity Forborne*** or ***in Arrears*** is one the payments of which were not made when due.

849. The ***Amount*** or ***Final Value*** of an annuity is the sum of all the payments increased by the interest of each payment from the time it becomes due until the annuity ceases.

850. The ***Present Worth*** of an annuity is such a sum of money as will, in the given time, and at the given rate per cent., amount to the final value.

851. An annuity is said to be *deferred* when it does not begin until after a certain period of time; it is said to be *reversionary* when it does not begin until after the occurrence of some specified future event, as the death of a certain person; and it is said to be *in possession* when it has begun, or begins immediately.

ANNUITIES AT SIMPLE INTEREST.

852. All problems in annuities at simple interest may be solved by combining the rules in Arithmetical Progression with those in Simple Interest.

WRITTEN EXERCISES.

853. 1. What is the amount of an annuity of $300 for 5 years, at 6 per cent. simple interest?

OPERATION.

$$\frac{300+372}{2} \times 5 = 1680$$

ANALYSIS.—At the end of the 5th year the following sums were due:

The 5th year's payment = $300,
The 4th year's payment = $300 + $18 = $318,
The 3d year's payment = $300 + $36 = $336,
The 2d year's payment = $300 + $54 = $354,
The 1st year's payment = $300 + $72 = $372.

These sums form an arithmetical progression, in which the first term is the annuity, $300, the common difference is the interest of the annuity for 1 year, and the number of terms is the number of years. The sum of all the terms of this progression is $1680 (**832**), which is the amount of the annuity.

2. A father deposits annually for the benefit of his son, beginning with his tenth birthday, such a sum that on his 21st birthday the first deposit, at simple int., amounts to $210, and the sum due his son is $1860. Find the annual deposit, and at what rate per cent. it is deposited.

OPERATION.

$6 \times (\text{1st term} + 210) = 1860.$ (**832.**)

Hence, $\text{1st term} = 310 - 210 = 100 = a.$

$(210 - 100) \div (12 - 1) = \frac{110}{11} = 10 = d.$ (**830.**)

ANALYSIS.—Here $210, the first deposit, is the last term; 12, the number of deposits, is the number of terms;

and, $1860, the final value of the annuity, is the sum of all the terms. Using the principle of **832**, we find the *first term* to be $100, which is the annual deposit. By **830**, the *common difference* is found to be $10; hence 10 per cent. is the required rate.

3. What is the amount of an annuity of $150 for 5½ years, payable quarterly, at 1½ per cent. per quarter?

4. What is the present worth of an annuity of $300 for 5 years, at 6 per cent.?

5. What is the present worth of an annuity of $500 for 10 years, at 10 per cent.?

6. In what time will an annual pension of $500 amount to $3450, at 6 per cent. simple interest?

7. Find the rate per cent. at which an annuity of $6000 will amount to $59760 in 8 years, at simple interest.

8. A man works for a farmer 1 yr. 6 mo., at $20 per month, payable monthly; and these wages remain unpaid until the expiration of the whole term of service. What is due the workman, allowing simple interest at 6 per cent. per annum?

ANNUITIES AT COMPOUND INTEREST.

854. All problems in annuities at compound interest may be solved by combining the rules in Geometrical Progression with those in Compound Interest.

WRITTEN EXERCISES.

1. What is the amount of an annuity of $300 for 5 years, at 6 per cent. compound interest?

OPERATION.

$$\frac{300 \times 1.06^5 - 300}{.06} = 1691.13$$

ANALYSIS.—At the end of the 5th year the following sums are due:

The 5th year's payment $= \$300$,
The 4th year's payment + interest for 1 year $= \$300 \times 1.06$,
The 3d year's payment + compound int. for 2 years $= \$300 \times 1.06^2$,
The 2d year's payment + compound int. for 3 years $= \$300 \times 1.06^3$,
The 1st year's payment + compound int. for 4 years $= \$300 \times 1.06^4$.

These sums form a geometrical progression, in which the first term is the annuity, $300, the ratio is the amount of $1 for 1 year, and the number of terms is the number of years. The sum of all the terms of this progression is $1691.13 (**843**), which is the amount of the annuity.

2. What is the present worth of an annuity of $300 for 5 years, at 6 per cent. compound interest?

OPERATION..

$$\frac{1691.13}{1.338226} = 1263.71$$

ANALYSIS.—The amount of this annuity is $1691.13. The amount of $1 for 5 years, at 6 per cent. compound interest, is $1.338226 (**587**). Hence the present worth of the annuity is $\frac{\$1691.13}{1.338226}$, or $1263.71.

3. Find the annuity whose amount for 25 years, at 6 per cent. compound interest, is $16459.35.

4. What is the present worth of an annuity of $700 for 7 years, at 6 per cent. compound interest?

5. An annuity of $200 for 12 years is in reversion 6 years. What is its present worth, compound interest at 6%?

6. A man bought a tract of land for $4800, which was to be paid in installments of $600 a year; how much money, at 6 per cent. compound interest, would discharge the debt at the time of the purchase?

7. What is the present value of a reversionary lease of $100, commencing 14 years hence, and to continue 20 years, compound interest at 5 per cent.?

855. SYNOPSIS FOR REVIEW.

INVOLUTION.
- 1. DEFS. 1. A Power. 2. Involution. 3. Base, or Root. 4. Exponent. 5. Square. 6. Cube. 7. Perfect Power.
- 2. PRINCIPLE.
- 3. **802.** RULE. 1. For Integers. 2. For Fractions.
- 4. **803.** 1. Principle. 2. Geometrical Illustration.
- 5. **804.** 1. Principle. 2. Geometrical Illustration.

EVOLUTION.
- 1. DEFS. 1. Square Root. 2. Cube Root, etc. 3. Evolution. 4. Radical Sign. 5. Index.
- 2. **810.** Rule.
- 3. **812.** Principles, 1, 2, 3, 4.
- 4. **813.** Rule, I, II, III. For Fractions.
- 5. **814.** Geometrical Illustration.
- 6. **818.** Principles, 1, 2, 3, 4.
- 7. **819.** Rule, I, II, III, IV, V, VI. For Fractions.
- 8. **820.** Geometrical Illustration.
- 9. **822.** Roots of a Higher Degree. *Rule.*

PROGRESSIONS.
- 1. Defs. 1. *Arithmetical Progression.* 2. *Terms.* 3. *Common Difference.* 4. *Increasing Arithmetical Progression.* 5. *Decreasing Arithmetical Progression.*
- 2. Quantities considered.
- 3. **829.** Rule, I, II. *Formulæ.*
- 4. **830.** Rule. *Formula.*
- 5. **831.** Rule. *Formula.*
- 6. **832.** Rule. *Formula.*
- 1. Defs. 1. *Geometrical Progression.* 2. *Terms.* 3. *Ratio.* 4. *Increasing Geom. Prog.* 5. *Decreasing Geom. Prog.* 6. *Infinite Decreasing Geom. Prog.*
- 2. Quantities considered.
- 3. **840.** Rule, I, II. *Formulæ.*
- 4. **841.** Rule. *Formula.*
- 5. **842.** Rule. *Formula.*
- 6. **843.** Rule. *Formula.*

ANNUITIES.
- 1. DEFS. 1. Annuity. 2. Certain Annuity. 3. Perpetuity. 4. Contingent Annuity. 5. Annuity in Arrears. 6. Amount. 7. Present Worth of an Annuity. 8. Deferred Annuity. 9. Reversionary Annuity. 10. Annuity in Possession.
- 3. ANNUITIES AT SIMPLE INTEREST.
- 2. ANNUITIES AT COMP. INTEREST.

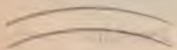
} Problems, how solved.

MENSURATION

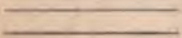
856. ***Mensuration*** is the process of finding the number of units in extension.

LINES.

857. A ***Straight Line*** is a line that does not change its direction. It is the shortest distance between two points.

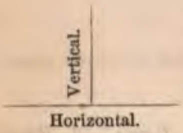


858. A ***Curved Line*** changes its direction at every point.



859. ***Parallel Lines*** have the same direction; and being in the same plane and equally distant from each other, they can never meet.

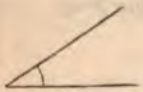
860. A ***Horizontal Line*** is a line parallel to either the horizon or water level.



861. A ***Perpendicular Line*** is a straight line drawn to meet another straight line, so as to incline no more to the one side than to the other.

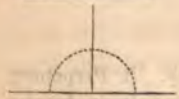
A perpendicular to a horizontal line is called a *vertical* line.

ANGLES.

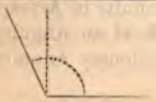


862. An ***Angle*** is the difference in the direction of two lines proceeding from a common point, called the *vertex.*

Angles are measured by degrees. (**301.**)



863. A ***Right Angle*** is an angle formed by two lines perpendicular to each other.



864. An ***Obtuse Angle*** is greater than a right angle.

865. An ***Acute Angle*** is less than a right angle.

All angles except right angles are called *oblique angles.*

PLANE FIGURES.

866. A ***Plane Figure*** is a portion of a plane surface bounded by straight or curved lines.

867. A ***Polygon*** is a plane figure bounded by straight lines.

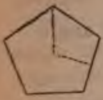
868. The ***Perimeter*** of a polygon is the sum of its sides.

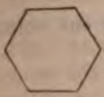
869. The ***Area*** of a plane figure is the surface included within the lines which bound it. (**460.**)

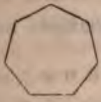
A *regular polygon* has all its sides and all its angles equal.

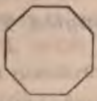
The *altitude* of a polygon is the perpendicular distance between its *base* and a side or angle opposite.

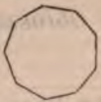
A polygon of three sides is called a *trigon*, or triangle; of four sides, a *tetragon*, or quadrilateral; of five sides, a *pentagon*, etc.

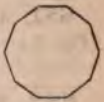

Pentagon.


Hexagon.


Heptagon.


Octagon.


Nonagon.

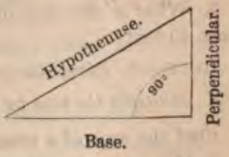

Decagon.

TRIANGLES.

870. A ***Triangle*** is a plane figure bounded by three sides, and having three angles.

871. A ***Right-Angled Triangle*** is a triangle having one right angle.

872. The ***Hypothenuse*** of a right-angled triangle is the side opposite the right angle.



873. The ***Base*** of a triangle, or of any plane figure, is the side on which it may be supposed to stand.

874. The ***Perpendicular*** of a right-angled triangle is the side which forms a right angle with the base.

875. The ***Altitude*** of a triangle is a line drawn from the angle opposite perpendicular to the base.

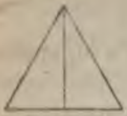
1. The dotted lines in the following figures represent the *altitude*.
2. Triangles are named from the relation both of their sides and angles.

876. An ***Equilateral Triangle*** has its three sides equal.

877. An ***Isosceles Triangle*** has only two of its sides equal.

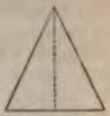
878. A ***Scalene Triangle*** has all of its sides unequal.

FIG. 1.



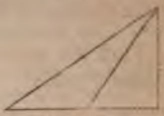
Equilateral.

FIG. 2.



Isosceles.

FIG. 3.



Scalene.

879. An ***Equiangular Triangle*** has three equal angles (Fig. 1.)

880. An ***Acute-angled Triangle*** has three acute angles. (Fig. 2.)

881. An ***Obtuse-angled Triangle*** has one obtuse angle. (Fig. 3.)

PROBLEMS.

882. The base and altitude of a triangle being given to find its area.

1. Find the area of a triangle whose base is 26 ft. and altitude 14.5 feet.

OPERATION.—$\overline{14.5\times26}\div2=188\frac{1}{2}$ sq. ft. Or, $26\times\frac{14.5}{2}=188\frac{1}{2}$ square feet, *area*.

2. What is the area of a triangle whose altitude is 10 yards and base 40 feet?

RULE.—1. *Divide the product of the base and altitude by* 2. Or,

2. *Multiply the base by one-half the altitude.*

Find the area of a triangle

3. Whose base is 12 ft. 6 in. and altitude 6 ft. 9 in.

4. Whose base is 25.01 chains and altitude 18.14 chains.

5. What is the cost of a triangular piece of land whose base is 15.48 ch. and altitude 9.67 ch., at $60 an acre?

6. At $.40 a square yard, find the cost of paving a triangular court, its base being 105 feet, and its altitude 21 yards?

7. Find the area of the gable end of a house that is 28 ft. wide, and the ridge of the roof 15 ft. higher than the foot of the rafters.

883. The area and one dimension being given to find the other dimension.

1. What is the base of a triangle whose area is 189 square feet and altitude 14 feet?

OPERATION.—$(189 \text{ sq. ft.} \times 2) \div 14 = 27$ ft., *base*.

2. Find the altitude of a triangle whose area is $20\frac{1}{4}$ square feet and base 3 yards.

RULE.—*Double the area, then divide by the given dimension.*

Find the other dimension of the triangle

3. When the area is 65 sq. in. and the altitude 10 inches.

4. When the base is 42 rods and the area 588 sq. rods.

5. When the area is $6\frac{1}{2}$ acres and the altitude 17 yards.

6. When the base is 12.25 chains and the area 5 A. 33 P.

7. Paid $1050 for a piece of land in the form of a triangle, at the rate of $\$5\frac{1}{4}$ per square rod. If the base is 8 rd., what is its altitude?

884. The three sides of a triangle being given to find its area.

1. Find the area of a triangle whose sides are 30, 40, and 50 ft.

OPERATION.—$(30+40+50) \div 2 = 60$; $60-30 = 30$; $60-40 = 20$; $60-50 = 10$. $\sqrt{60 \times 30 \times 20 \times 10} = 600$ ft., *area*.

2. What is the area of an isosceles triangle whose base is 20 ft., and each of its equal sides 15 feet?

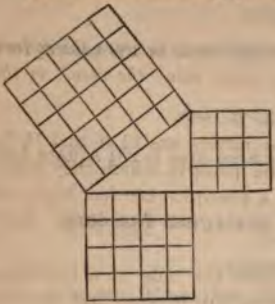
RULE.—*From half the sum of the three sides, subtract each side separately; multiply the half-sum and the three remainders together; the square root of the product is the area.*

3. Find the area of a triangle whose sides are 25, 36, and 49 in.

4. How many acres in a field in the form of an equilateral triangle whose sides each measure 70 rods?

5. The roof of a house 30 ft. wide has the rafters on one side 20 ft. long, and on the other 18 ft. long. How many square feet of boards will be required to board up both gable ends?

885. The following principles relating to *right-angled* triangles have been established by Geometry:



PRINCIPLES.—1. *The square of the hypothenuse of a right-angled triangle is equal to the sum of the squares of the other two sides.*

2. *The square of the base, or of the perpendicular, of a right-angled triangle is equal to the square of the hypothenuse diminished by the square of the other side.*

886. To find the hypothenuse.

1. The base of a right-angled triangle is 12, and the perpendicular 16. What is the length of the hypothenuse?

OPERATION.—$12^2 + 16^2 = 400$ (PRIN. 1). $\sqrt{400} = 20$, *hypothenuse.*

2. The foot of a ladder is 15 feet from the base of a building, and the top reaches a window 36 feet above the base. What is the length of the ladder?

RULE.—*Extract the square root of the sum of the squares of the base and the perpendicular; the result is the hypothenuse.*

3. If the gable end of a house 40 ft. wide is 16 ft. high, what is the length of the rafters?

4. A park 25 chains long and 23 chains wide has a walk running through it from opposite corners in a straight line. What is the length of the walk?

5. A room is 20 ft. long, 16 ft. wide, and 12 ft. high. What is the distance from one of the lower corners to the opposite upper corner?

887. To find the base or perpendicular.

1. The hypothenuse of a right-angled triangle is 35 feet, and the perpendicular 28 feet. Find the base.

OPERATION.—$35^2 - 28^2 = 441$ (PRIN. 2). $\sqrt{441} = 21$ ft., *base.*

2. The hypothenuse of a right-angled triangle is 53 yards and the base 84 feet. Find the perpendicular.

RULE.—*Extract the square root of the difference between the square of the hypothenuse and the square of the given side; the result is the required side.*

3. Find the width of a house, whose rafters are 12 ft. and 15 ft. long, and that form a right angle at the point in which they meet.

4. A line reaching from the top of a precipice 120 feet high, on the bank of a river, to the opposite side is 380 feet long. How wide is the river?

5. A ladder 52 ft. long stands against the side of a building. How many feet must it be drawn out at the bottom that the top may be lowered 4 feet?

QUADRILATERALS.

888. A ***Quadrilateral*** is a plane figure bounded by four straight lines.

There are three kinds of quadrilaterals, the *Parallelogram*, *Trapezoid*, and *Trapezium*.

889. A ***Parallelogram*** is a quadrilateral which has its opposite sides parallel.

There are four kinds of parallelograms, the *Square*, *Rectangle*, *Rhomboid*, and *Rhombus*.

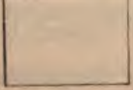
890. A ***Rectangle*** is any parallelogram having its angles right angles.

891. A ***Square*** is a rectangle whose sides are equal.

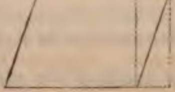
892. A ***Rhomboid*** is a parallelogram whose opposite sides *only* are equal, and whose angles are not right angles.

893. A ***Rhombus*** is a parallelogram whose sides are *all* equal, but whose angles are not right angles.

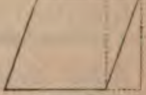
Square.



Rectangle.



Rhomboid.



Rhombus.

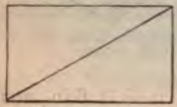
894. A ***Trapezoid*** is a quadrilateral, two of whose sides are *parallel.*

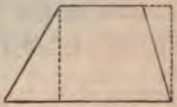
895. A ***Trapezium*** is a quadrilateral having no two sides parallel.

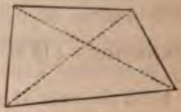
896. The ***Altitude*** of a parallelogram or trapezoid is the perpendicular distance between its *parallel* sides.

The dotted vertical lines in the figure represent the *altitude.*

897. A ***Diagonal*** of a plane figure is a straight line joining the vertices of two angles not adjacent.


Parallelogram.


Trapezoid.


Trapezium.

PROBLEMS.

898. To find the area of any parallelogram.

1. Find the area of a parallelogram whose base is 16.25 feet and altitude 7.5 feet.

OPERATION.—16.25 ft. $\times 7.5 = 121.875$ sq. feet, *area.*

2. The base of a rhombus is 10 feet 6 inches, and its altitude 8 feet. What is its area?

RULE.—*Multiply the base by the altitude.*

3. How many acres in a piece of land in the form of a rhomboid, the base being 8.75 ch. and altitude 6 chains?

899. To find the area of a trapezoid.

1. Find the area of a trapezoid whose parallel sides are 23 and 11 feet, and the altitude 9 feet.

OPERATION.—$\overline{23 \text{ ft.} + 11 \text{ ft.}} \div 2 = 17$ ft.; 17 ft. $\times 9 = 153$ sq. ft., *area.*

2. Required the area of a trapezoid whose parallel sides are 178 and 146 feet, and the altitude 69 feet.

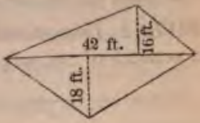
RULE.—*Multiply one-half the sum of the parallel sides by the altitude.*

3. How many square feet in a board 16 ft. long, 18 inches wide at one end and 25 inches wide at the other end?

4. One side of a quadrilateral field measures 38 rods; the side opposite and parallel to it measures 26 rods, and the distance between the two sides is 10 rods. Find the area.

900. To find the area of a trapezium.

1. Find the area of a trapezium whose diagonal is 42 feet and perpendiculars to this diagonal, as in the diagram, are 16 feet and 18 feet.



OPERATION.—$(\overline{18\text{ ft.}+16\text{ ft.}}\div 2)\times 42 = 714$ sq. feet, *area*.

2. Find the area of a trapezium whose diagonal is 35 ft. 6 in., and the perpendiculars to this diagonal 9 feet and 3 feet.

RULE.—*Multiply the diagonal by half the sum of the perpendiculars drawn to it from the vertices of opposite angles.*

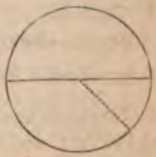
3. How many acres in a quadrilateral field whose diagonal is 80 rd. and the perpendiculars to this diagonal 20.453 and 50.832 rd.?

To find the area of any *regular* polygon, multiply its perimeter, or the sum of its sides, by the perpendicular falling from its center to one of its sides.

To find the area of an *irregular* polygon, divide the figure into triangles and trapeziums, and find the area of each separately. The sum of these areas will be the area of the whole polygon.

THE CIRCLE.

901. A ***Circle*** is a plane figure bounded by a curved line, called the *circumference*, every point of which is equally distant from a point within called the *center*.



902. The ***Diameter*** of a circle is a line passing through its center, and terminated at both ends by the circumference.

903. The ***Radius*** of a circle is a line extending from its center to any *point in the* circumference. It is one-half the diameter.

PROBLEMS.

904. When either the diameter or the circumference of a circle is given, to find the other dimension of it.

1. Find the circumference of a circle whose diameter is 20 inches.

OPERATION.—20 in. × 3.1416 = 62.832 in. = 5 ft. 2.832 in., *circum.*

2. Find the diameter of a circle whose circumference is 62.832 ft.

OPERATION.—62.832 ft. ÷ 3.1416 = 20 ft., *diameter.*

3. Find the diameter of a wheel whose circumference is 50 feet.

RULE.—1. *Multiply the diameter by* 3.1416 ; *the product is the circumference.*

2. *Divide the circumference by* 3.1416 ; *the quotient is the diameter.*

4. What is the diameter of a tree whose girt is 18 ft. 6 in.?

5. What is the radius of a circle whose circumference is 31.416 ft.?

6. Find the circumference of the greatest circle that can be drawn with a string 14 inches long, used as a radius.

905. To find the area of a circle, when both its diameter and circumference are given, or when either is given.

1. What is the area of a circle whose diameter is 10 feet and circumference 31.416 feet?

OPERATION.—$31.416 \text{ ft.} \times \overline{10 \div 4} = 78.54$ sq. ft., *area.*

2. Find the area of a circle whose diameter is 10 feet.

OPERATION.—$10 \text{ ft.}^2 \times .7854 = 78.54$ sq. feet, *area.*

3. Find the area of a circle whose circumference is 31.416 feet.

OPERATION.—31.416 ft. ÷ 3.1416 = 10 ft., *diam.*; $(10 \text{ ft.})^2 \times .7854 =$ 78.54 sq. feet, *area.*

RULES.—To find the area of a circle :

1. *Multiply* ¼ *of its diameter by the circumference.*

2. *Multiply the square of its diameter by* .7854.

4. What is the area of a circular pond whose circumference is 200 chains

5. The distance around a circular park is 1½ miles. How many acres does it contain?

906. To find the diameter or circumference of a circle, when the area is given.

1. What is the diameter of a circle whose area is 1319.472?

OPERATION.—$1319.472 \div .7854 = 1680$; $\sqrt{1680} = 40.987+$, *diameter.*

2. What is the circumference of a circle whose area is 19.635?

OPERATION—$19.635 \div 3.1416 = 6.25$; $\sqrt{6.25} = 2.5$, *radius*; $2.5 \times 2 \times 3.1416 = 15.708$, *circumference.*

RULE.—1. *Divide the area by* .7854 *and extract the square root of the quotient; the result is the diameter.*

2. *Divide the area by* 3.1416 *and extract the square root of the quotient; the result is the radius. The circumference is obtained by Art.* **904,** 1. Or,

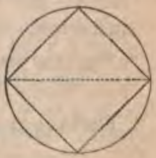
3. *Divide the area by* .07958 *and find the square root of the quotient.*

3. The area of a circular lot is 38.4846 square rods. What is its diameter?

4. The area of a circle is 286.488 square feet. Required the diameter and the circumference.

907. To find the side of an inscribed square when the diameter of the circle is known.

1. What is the side of a square inscribed in a circle whose diameter is 6 rods?



OPERATION.—$6^2 \div 2 = 18$; $\sqrt{18} = 4.24$ rods, *side of square.*

2. The diameter of a circle is 200 feet. Find the side of the inscribed square.

RULE.—1. *Extract the square root of half the square of the diameter.* Or,

2. *Multiply the diameter by* .7071.

3. The circumference of a circle is 104 yards. Find the side of the inscribed square.

4. The area of a circle is 78.54 square feet. Find the side of the inscribed square.

908. To find the area of a circular ring formed by two concentric circles.

1. Find the area of a circular ring, when the diameters of the circles are 20 and 30 feet.

Operation.—$(\overline{30 + 20} \times \overline{30 - 20}) \times .7854 =$ 392.7 sq. ft., *area.*

2. Find the area of a circular ring formed by two concentric circles, whose diameters are 7 ft. 9 in. and 4 ft. 3 in.

Rule.—*Multiply the sum of the two diameters by their difference, and the product by .7854; the result is the area.*

3. Two diameters are 35.75 and 16.25 ft.; find the area of the ring.

4. The area of a circle is 1 A. 154.16 P. In the center is a pond of water 10 rd. in diameter; find the area of the land and of the water.

909. To find a mean proportional between two numbers.

1. What is a mean proportional between 3 and 12?

Operation.—$\sqrt{12 \times 3} = 6$, the *mean proportional.*

When three numbers are proportional, the product of the extremes is equal to the square of the mean.

Rule.—*Extract the square root of the product of the two numbers.*

Find a mean proportional between

2. 42 and 168. | 3. 64 and 12.25. | 4. $\frac{36}{49}$ and $\frac{4}{81}$.

5. A tub of butter weighed 36 lb. by the grocer's scales; but weighing it in the other scale of the balance, it weighed only 30 pounds. What was the true weight of the butter?

SIMILAR PLANE FIGURES.

910. ***Similar Plane Figures*** are such as have the same *form*, viz., equal angles, and their like dimensions proportional.

All circles, squares, equiangular triangles, and regular polygons of the same number of sides are similar figures.

The like dimensions of circles are their radii, diameters, and circumferences.

Principles.—1. *The like dimensions of similar plane figures are proportional.*

2. *The areas of similar plane figures are to each other as the squares of their like dimensions.* And conversely,

3. *The like dimensions of similar plane figures are to each other as the square root of their areas.*

The same principles apply also to the *surfaces* of all similar figures, such as triangles, rectangles, etc.; the surfaces of similar *solids*, as cubes, pyramids, etc.; and to similar *curved surfaces*, as of cylinders, cones, and spheres. Hence,

4. *The surfaces of all similar figures are to each other as the squares of their like dimensions.* And conversely,

5. *Their dimensions are as the square roots of their surfaces.*

PROBLEMS.

1. A triangular field whose base is 12 ch. contains 2 A. 80 P. Find the area of a field of similar form whose base is 48 chains.

Operation.—$12^2 : 48^2 :: 2$ A. 80 P. : x P. $= 6400$ P. $= 40$ A., *area*. (Prin. 2.)

2. The side of a square field containing 18 acres is 60 rods long. Find the side of a similar field that contains $\frac{1}{3}$ as many acres.

Operation.—18 A. : 6 A. :: $60^2 : x^2 = 1200$; $\sqrt{1200} = 34.64$ rd. +, *side*. (Prin. 3.)

3. Two circles are to each other as 9 to 16; the diameter of the less being 112 feet, what is the diameter of the greater?

Operation.—$9 : 16 :: 112^2 : x^2 = 3 : 4 :: 112 : x = 149$ ft. 4 in., *diameter*. (Prin. 2.)

4. A peach orchard contains 720 square rods, and its length is to its breadth as 5 to 4; what are its dimensions?

Operation.—The area of a rectangle 5 by 4 equals 20 (**898**).

$20 : 720 :: 5^2 : x^2 = 900$; $\sqrt{900} = 30$ rd., *length*.

$20 : 720 :: 4^2 : x^2 = 576$; $\sqrt{576} = 24$ rd., *width*.

5. It is required to lay out 283 A. 107 P. of land in the form of a rectangle, so that the length shall be 3 times the width. Find the dimensions.

6. A pipe 1.5 in. in diameter fills a cistern in 5 hours; find the diameter of a pipe that will fill the same cistern in 55 min. 6 sec.

7. The area of a triangle is 24276 sq. ft., and its sides in proportion to the numbers 13, 14, and 15. Find the length of its sides in feet.

8. If it cost $167.70 to enclose a circular pond containing 17 A. 110 P., what will it cost to enclose another ⅕ as large?

9. If 63.39 rods of fence will enclose a circular field containing 2 acres, what length will enclose 8 acres in circular form?

REVIEW OF PLANE FIGURES.

PROBLEMS.

911. 1. How much less will the fencing of 20 acres cost in the square form than in the form of a rectangle whose breadth is ⅕ the length, the price being $2.40 per rod?

2. A house that is 50 feet long and 40 feet wide has a square or pyramidal roof, whose height is 15 ft. Find the length of a rafter reaching from a corner of the building to the vertex of the roof.

3. Find the diameter of a circular island containing 1¼ sq. miles.

4. What is the value of a farm, at $75 an acre, its form being a quadrilateral, with two of its opposite sides parallel, one 40 ch. and the other 22 ch. long, and the perpendicular distance between them 25 chains?

5. Find the cost, at 18 cents a square foot, of paving a space in the form of a rhombus, the sides of which are 15 feet, and a perpendicular drawn from one oblique angle will meet the opposite side 9 feet from the adjacent angle.

6. A goat is fastened to the top of a post 4 ft. high by a rope 50 ft. long. Find the area of the greatest circle over which he can graze.

7. How much larger is a square circumscribing a circle 40 rods in diameter, than a square inscribed in the same circle?

8. What is the value of a piece of land in the form of a triangle, whose sides are 40, 48, and 54 rods, respectively, at the rate of $125 an acre?

9. The radius of a circle is 5 feet; find the diameter of another circle containing 4 times the area of the first.

10. How many acres in a semi-circular farm, whose radius is 100 rods?

11. What must be the width of a walk extending around a garden 100 feet square, to occupy one-half the ground?

12. An irregular piece of land, containing 540 A. 36 P. is exchanged for a square piece of the same area; find the length of one of its sides? If divided into 42 equal squares, what is the length of the side of each?

13. A field containing 15 A. is 30 rd. wide, and is a plane inclining in the direction of its length, one end being 120 ft. higher than the other. Find how many acres of horizontal surface it contains.

14. If a pipe 3 inches in diameter discharges 12 hogsheads of water in a certain time, what must be the diameter of a pipe which will discharge 48 hogsheads in the same time?

SOLIDS.

912. A ***Solid*** or ***Body*** has three dimensions, length, breadth, and thickness.

The planes which bound it are called its *faces*, and their intersections, its *edges*.

913. A ***Prism*** is a solid whose ends are equal and parallel, similar polygons, and its sides parallelograms.

Prisms take their names from the *form* of their bases, as *triangular*, *quadrangular*, *pentagonal*, etc.

914. The ***Altitude*** of a prism is the perpendicular distance between its bases.

Parallelopipedon.

915. A ***Parallelopipedon*** is a prism bounded by six parallelograms, the opposite ones being parallel and equal.

Cube.

916. A ***Cube*** is a parallelopipedon whose faces are all equal squares.

917. A ***Cylinder*** is a body bounded by a uniformly curved surface, its ends being equal and parallel circles.

1. A cylinder is conceived to be generated by the revolution of a rectangle about one of its sides as an axis.

2. The line joining the centers of the bases, or ends, of the cylinder is its *altitude*, or *axis*.

Triangular Prism.

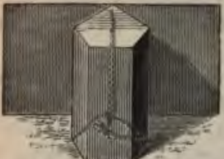
Quadrangular Prism.

Pentagonal Prism.

Cylinder.

PROBLEMS.

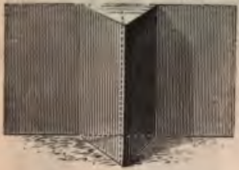
918. To find the convex surface of a prism or cylinder.



1. Find the area of the convex surface of a prism whose altitude is 7 ft., and its base a pentagon, each side of which is 4 feet.

OPERATION.—4 ft. × 5 = 20 ft., *perimeter.*

20 ft. × 7 = 140 sq. ft., *convex surface.*

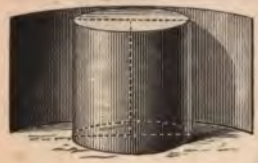


2. Find the area of the convex surface of a triangular prism, whose altitude is $8\frac{1}{2}$ feet, and the sides of its base 4, 5, and 6 feet, respectively.

OPERATION.—4 ft. + 5 ft. + 6 ft. = 15 ft., *perimeter.*

15 ft. × $8\frac{1}{2}$ = $127\frac{1}{2}$ sq. ft., *convex surface.*

3. Find the area of the convex surface of a cylinder whose altitude is 2 ft. 5 in. and the circumference of its base 4 ft. 9 in.



OPERATION.—2 ft. 5 in. = 29 in.; 4 ft. 9 in. = 57 in.

57 in. × 29 = 1653 sq. in. = 11 sq. ft. 69 sq. inches, *convex surface.*

RULE.—*Multiply the perimeter of the base by the altitude.*

To find the *entire* surface, add the area of the bases or ends.

4. If a gate 8 ft. high and 6 ft. wide revolves upon a point in its center, what is the entire surface of the cylinder described by it?

5. Find the superficial contents, or entire surface of a parallelopipedon 8 ft. 9 in. long, 4 ft. 8 in. wide, and 3 ft. 3 in. high.

6. What is the entire surface of a cylinder formed by the revolution about one of its sides of a rectangle that is 6 ft. 6 in. long and 4 ft. wide?

7. Find the entire surface of a prism whose base is an equilateral *triangle,* the perimeter being 18 ft., and the altitude 15 ft.

919. To find the volume of any prism or cylinder.

1. Find the volume of a triangular prism, whose altitude is 20 ft., and each side of the base 4 feet.

OPERATION.—The area of the base is 6.928 sq. ft. (**882**).
6.928 sq. ft. × 20 = 138.56 cu. ft., *volume*.

2. Find the volume of a cylinder whose altitude is 8 ft. 6 in., and the diameter of its base 3 feet.

OPERATION.—$3^2 \times .7854 = 7.0686$ square feet, *area of base* (**905**).
7.0686 sq. ft. × 8.5 = 60.083 cubic feet, *volume*.

RULE.—*Multiply the area of the base by the altitude.*

3. Find the solid contents of a cube whose edges are 6 ft. 6 in.

4. Find the cost of a piece of timber 18 in. square and 40 ft. long, at $.30 a cubic foot.

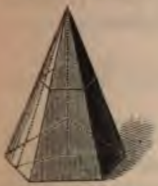
5. Required the solid contents of a cylinder whose altitude is 15 ft. and its radius 1 ft. 3 in.

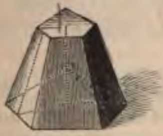
6. What is the value of a log 24 ft. long, of the average circumference of 7.9 ft., at $.45 a cubic foot?

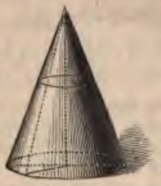
PYRAMIDS AND CONES.

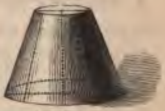
920. A ***Pyramid*** is a body having for its base a polygon, and for its other faces three or more triangles, which terminate in a common point called the *vertex*.

Pyramids, like prisms, take their names from their bases, and are called *triangular*, *square*, or *quadrangular*, *pentagonal*, etc.


Pyramid.


Frustum.


Cone.


Frustum.

921. A ***Cone*** is a body having a circular base, and whose convex surface tapers uniformly to the *vertex*.

It is a body conceived to be formed by the revolution of a right-angled triangle about one of its sides containing the right angle, as an immovable axis.

922. The ***Altitude*** of a pyramid or of a cone is the perpendicular *distance from its* vertex to the plane of its base.

923. The ***Slant Height*** of a *pyramid* is the perpendicular distance from its vertex to one of the sides of the base; of a *cone*, is a straight line from the vertex to the circumference of the base.

924. The ***Frustum*** of a pyramid or cone is that part which remains after cutting off the top by a plane parallel to the base.

PROBLEMS.

925. To find the convex surface of a pyramid or cone.

1. Find the convex surface of a triangular pyramid, the slant height being 16 ft., and each side of the base 5 feet.

OPERATION.—$(5\text{ ft.}+5\text{ ft.}+5\text{ ft.})\times\overline{16\div2}=120$ sq. ft., *conv. surf.*

2. Find the convex surface of a cone whose diameter is 17 ft. 6 in., and the slant height 30 feet.

RULE.—*Multiply the perimeter or circumference of the base by one-half the slant height.*

To find the *entire* surface, add to this product the area of the base.

3. Find the entire surface of a square pyramid whose base is 8 ft. 6 in. square, and its slant height 21 feet.

4. Find the entire surface of a cone the diameter of whose base is 6 ft. 9 in. and the slant height 45 ft.

5. Find the cost of painting a church spire, at $.25 a sq. yd., whose base is a hexagon 5 ft. on each side, and the slant height 60 feet.

926. To find the volume of a pyramid or of a cone.

1. What is the volume, or solid contents, of a square pyramid whose base is 6 feet on each side, and its altitude 12 feet.

OPERATION.—$\overline{6\times6}\times\overline{12\div3}=144$ cu. ft., *volume.*

2. Find the volume of a cone, the diameter of whose base is 5 ft. and its altitude $10\frac{1}{2}$ feet.

OPERATION.—$(5^2\text{ ft.}\times.7854)\times\overline{10\frac{1}{2}\div3}=68.72\frac{1}{4}$ cu. ft., *volume.*

RULE.—*Multiply the area of the base by one-third the altitude.*

3. Find the solid contents of a cone whose altitude is 24 ft., and the diameter of its base 30 inches.

4. What is the cost of a triangular pyramid of marble, whose altitude is 9 ft., each side of the base being 3 ft., at $$2\frac{1}{2}$ per cu. foot?

5. Find the volume and the entire surface of a pyramid whose base is a rectangle 80 feet by 60 feet, and the edges which meet at the vertex are 130 feet.

927. To find the convex surface of a frustum of a pyramid or cone.

1. What is the convex surface of a frustum of a square pyramid, whose slant height is 7 feet, each side of the greater base 4 feet, and of the less base 18 inches?

OPERATION.—The *perimeter* of the greater base is 16 ft., of the less 6 feet.

$$\overline{16\text{ ft.}+6\text{ ft.}} \times \overline{7 \div 2} = 77\text{ sq. ft., } \textit{convex surface.}$$

2. Find the convex surface of a frustum of a cone whose slant height is 15 feet, the circumference of the lower base 30 feet, and of the upper base 16 feet.

RULE.—*Multiply the sum of the perimeters, or of the circumferences, by one-half the slant height.*

To find the *entire* surface, add to this product the area of both ends, or bases.

3. How many square yards in the convex surface of a frustum of a pyramid, whose bases are heptagons, each side of the lower base being 8 feet, and of the upper base 4 feet, and the slant height 55 feet?

928. To find the volume of a frustum of a pyramid or cone.

1. Find the volume of the frustum of a square pyramid whose altitude is 10 feet, each side of the lower base 12 feet, and of the upper base 9 feet.

OPERATION.—$12^2+9^2=225$; $(225+\sqrt{144\times 81})\times\overline{10\div 3}=1000$ cu. feet, *volume.*

2. How many cubic feet in the frustum of a cone whose altitude is 6 ft. and the diameters of its bases 4 ft. and 3 feet?

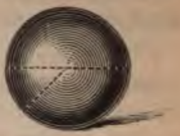
RULE.—*To the sum of the areas of both bases add the square root of the product, and multiply this sum by one-third of the altitude.*

3. How many cubic feet in a piece of timber 30 ft. long, the greater end being 15 inches square, and that of the less 12 inches?

4. How many cubic feet in the mast of a ship, its height being 50 ft., the circumference at one end 5 feet and at the other 3 feet?

THE SPHERE.

929. A ***Sphere*** is a body bounded by a uniformly curved surface, all the points of which are equally distant from a point within called the *center*.



930. The ***Diameter*** of a sphere is a straight line passing through the center of the sphere, and terminated at both ends by its surface.

931. The ***Radius*** of a sphere is a straight line drawn from the center to any point in the surface.

932. To find the surface of a sphere.

1. Find the surface of a sphere whose diameter is 9 in.

OPERATION.—9 in. × 3.1416 = 28.2744 in., *circumference.*
28.2744 in. × 9 = 254.4696 sq. in., *surface.*

RULE.—*Multiply the diameter by the circumference of a great circle of the sphere.*

2. What is the surface of a globe 3 feet in diameter?
3. Find the surface of a globe whose radius is 1 foot.

933. To find the volume of a sphere.

1. Find the volume of a sphere whose diameter is 18 inches.

OPERATION.—18 in. × 3.1416 = 56.5488 in., *circumference.*
56.5488 in. × 18 = 1017.8784 sq. in., *surface.*
1017.8784 sq. in. × $\overline{18 \div 6}$ = 3053.6352 cu. in., *volume.*

RULE.—*Multiply the surface by $\frac{1}{6}$ of the diameter, or $\frac{1}{3}$ of the radius.*

2. Find the volume of a globe whose diameter is 30 in.
3. Find the solid contents of a globe whose radius is 5 yards.

934. To find the three dimensions of a rectangular solid, the volume and the ratio of the dimensions being given.

1. What are the dimensions of a rectangular solid, whose volume is 4480 cu. ft., and its dimensions are to each other as 2, 5, and 7?

OPERATION.—$\sqrt[3]{4480 \div (2 \times 5 \times 7)} = 4$; 4 ft. × 2 = 8 ft., *height*; *4 ft. × 5 = 20* ft., *width*; 4 ft. × 7 = 28 ft., length.

RULE.—I. *Divide the volume by the product of the terms proportional to the three dimensions, and extract the cube root of the quotient.*

II. *Multiply the root thus obtained by each proportional term; the products will be the corresponding sides.*

2. What are the dimensions of a rectangular box whose volume is 3000 cu. ft., and its dimensions are to each other as 2, 3, and 4?

3. A pile of bricks in the form of a parallelopiped contains 30720 cu. feet, and the length, breadth, and height are to each other as 3, 4, and 5. What are the dimensions of the pile?

SIMILAR SOLIDS.

935. *Similar Solids* are such as have the same *form*, and differ from each other only in volume.

PRINCIPLES.—1. *The volumes of similar solids are to each other as the cubes of their like dimensions.*

1. If the volume of a ball 3 inches in diameter is 27 cu. in., what is the volume of a ball 7 inches in diameter.

OPERATION.—$3^3 : 7^3 :: 27$ cu. in. $: x = 343$ cu. in., *volume.*

2. *The like dimensions of similar solids are to each other as the cube roots of their volumes.*

3. If the diameter of a ball whose volume is 27 cu. in. is 3 in., what is the diameter of a ball whose volume is 343 cu. inches?

OPERATION.—$27 : 343 :: 3^3 : x^3 = 343$; $\sqrt[3]{343} = 7$ in., *diameter.*

REVIEW OF SOLIDS.

PROBLEMS.

936. 1. What is the edge of a cube whose entire surface is 1050 sq. feet, and what is its volume?

2. What must be the inner edge of a cubical bin to hold 1250 bu. of wheat?

3. How many gallons will a cistern hold, whose depth is 7 ft., the bottom being a circle 7 feet in diameter and the top 5 feet in diameter?

4. What is the value of a stick of timber 24 ft. long, the larger end being 15 in. square, and the less 6 in., at 28 cents a cubic foot?

5. If a cubic foot of iron were formed into a bar ½ an inch square, without waste, what would be its length?

6. If a marble column 10 in. in diameter contains 27 cu. ft., what is the diameter of a column of equal length that contains 81 cu. ft.?

7. How many board feet in a post 11 ft. long, 9 in. square at the bottom, and 4 in. square at the top?

8. The surface of a sphere is the same as that of a cube, the edge of which is 12 in. Find the volume of each.

9. A ball 4.5 in. in diameter weighs 18 oz. Avoir.; what is the weight of another ball of the same density, that is 9 in. in diameter?

10. In what time will a pipe supplying 6 gal. of water a minute fill a tank in the form of a hemisphere, that is 10 ft. in diameter?

11. The diameter of a cistern is 8 feet; what must be its depth to contain 75 hhd. of water?

12. How many bushels in a heap of grain in the form of a cone, whose base is 8 ft. in diameter and altitude 4 feet?

GAUGING.

937. ***Gauging*** is the process of finding the capacity or volume of casks and other vessels.

A cask is equivalent to a cylinder having the same length and a diameter equal to the *mean diameter* of the cask.

To find the *mean diameter* of a cask (*nearly*),

Add to the head diameter ⅔, or, if the staves are but little curved, .6, of the difference between the head and bung diameters.

To find the *volume* of a cask in gallons,

Multiply the square of the mean diameter by the length (both in inches) and this product by .0034.

1. How many gallons in a cask whose head diameter is 24 inches, bung diameter 30 in., and its length 34 inches?

OPERATION.—$24 + (\overline{30 - 24} \times \frac{2}{3}) = 28$ in., *mean diameter.*

$28^2 \times 34 \times .0034 = 90.63$ gal., *capacity.*

2. What is the volume of a cask whose length is 40 inches, the diameters 21 and 30 in., respectively.

3. How many gallons in a cask of slight curvature, 3 ft. 6 in. long, the head diameter being 26 in., the bung diameter 31 in.?

938. CIRCLES.

1. The ***Diameter***	× 3.1416	= *the circumference.*
	÷ .3183	
	× .8862	= *the side of an equal square.*
	÷ 1.1284	
	× .8660	= *the side of an inscribed equilateral triangle.*
	÷ .1547	
	× .7070	= *the side of an inscribed square.*
	÷ 1.4142	
2. The ***Circumference***	× .15915	= *the diameter.*
	÷ 6.28318	
	× .3183	= *the side of an equal square.*
	÷ 3.1416	
	× .2821	= *the side of an inscribed equilateral triangle.*
	÷ 3.5450	
	× .2756	= *the side of an inscribed square.*
	÷ 3.6276	
	× .2251	= *the radius.*
	÷ 4.4428	
3. The ***Area***	÷ 3.1416	= *the radius.*
	× 1.2732	= *the square of the diameter.*
	÷ .7854	
	× 12.5663	= *the sq're of the circumference.*
	÷ .07958	

939. SPHERES.

1. The ***Surface***	=	*Circumference* × *its diam.*
		$\textit{Radius}^2 \times 12.5664.$
		$\textit{Diameter}^2 \times 3.1416.$
		$\textit{Circumference}^2 \times .3183.$
2. The ***Volume***	=	$\textit{Surface} \times \frac{1}{6}$ *its diameter.*
		$\textit{Radius}^3 \times 4.1888.$
		$\textit{Diameter}^3 \times .5236.$
		$\textit{Circumference}^3 \times .0169.$
3. The ***Diameter***	=	$\sqrt{\textit{Of surface}} \times .5642.$
		$\sqrt[3]{\textit{Of volume}} \times 1.2407.$
4. The ***Circumference***	=	$\sqrt{\textit{Of surface}} \times 1.77255.$
		$\sqrt[3]{\textit{Of volume}} \times 3.8978.$
5. The ***Radius***	=	$\sqrt{\textit{Of surface}} \times .2821.$
		$\sqrt[3]{\textit{Of volume}} \times .6204.$
6. The ***Side of Inscribed Cube***	=	Radius × 1.1547.
		Diameter × .5774.

METRIC SYSTEM.

940. The ***Metric System*** of weights and measures is based upon the *decimal scale.*

941. The ***Meter*** is the *base* of the system, and is the *one ten-millionth* part of the distance on the earth's surface from the equator to either pole, or 39.37079 inches.

942. From the *Meter* are made the *Are* (air), the *Stere* (stair), the *Liter* (leeter), and the *Gram ;* these constitute the *primary* or *principal* units of the system, from which all the others are derived.

943. The ***Multiple Units,*** or higher denominations, are named by prefixing to the name of the *primary* units the Greek numerals, *Deka* (10), *Hecto* (100), *Kilo* (1000), and *Myra* (10000).

944. The ***Sub-multiple Units,*** or lower denominations, are named by prefixing to the names of the *primary* units the Latin numerals, *Deci* ($\frac{1}{10}$), *Centi* ($\frac{1}{100}$), *Milli* $\frac{1}{1000}$).

Hence, it is apparent from the *name* of a unit whether it is *greater* or *less* than the standard unit, and also *how many times.*

MEASURES OF EXTENSION.

945. The ***Meter*** is the *unit of length,* and is equal to 39.37 in., nearly.

TABLE.

Metric Denominations.				U. S. Value.
		1 Millimeter	=	.03937079 in.
10 Millimeters,	*mm.*	= 1 Centimeter	=	.3937079 in.
10 Centimeters,	*cm.*	= 1 Decimeter	=	3.937079 in.
10 Decimeters,	*dm.*	= 1 ***Meter***	=	39.37079 in.
10 METERS,	*M.*	= 1 Dekameter	=	32.808992 ft.
10 Dekameters,	*Dm.*	= 1 Hectometer	=	19.927817 rd.
10 Hectometers,	*Hm.*	= 1 Kilometer	=	.6213824 mi.
10 Kilometers,	*Km.*	= 1 Myriameter (Mm.)	=	6.213824 mi.

The ***Meter,*** like our yard, is used in measuring cloths and short distances.

The ***Kilometer*** is commonly used for measuring long distances, and is about $\frac{5}{8}$ of a common mile.

946. The ***Are*** is the *unit of land measure,* and is a square whose side is 10 meters, equal to a *square dekameter,* or 119.6 sq. yards.

TABLE.

1 Centiare, *ca.* = (1 Sq. Meter) = 1.196034 sq. yd.
100 Centiares, " = 1 ***Are*** = 119.6034 sq. yd.
100 ARES, *A.* = 1 Hectare (*Ha.*) = 2.47114 acres.

947. The ***Square Meter*** is the *unit* for measuring ordinary surfaces; as flooring, ceilings, etc.

TABLE.

100 Sq. Millimeters, *sq.mm.* = 1 Sq. Centimeter = .155 + sq. in.
100 Sq. Centimeters, *sq. cm.* = 1 Sq. Decimeter = 15.5 + sq. in.
100 Sq. Decimeters, *sq. dm.* = 1 ***Sq. Meter*** (*Sq.M.*) = 1.196 + sq. yd.

948. The ***Stere*** is the *unit* of wood or solid measure, and is equal to a *cubic meter*, or .2759 cord.

TABLE.

1 Decistere = 3.531 + cu. ft.
10 Decisteres, *dst.* = 1 ***Stere*** = 35.316 + cu. ft.
10 STERES, *St.* = 1 Dekastere (*DSt.*) = 13.079 + cu. yd.

949. The ***Cubic Meter*** is the *unit* for measuring ordinary solids; as excavations, embankments, etc.

TABLE.

1000 Cu. Millimeters, *cu. mm.* = 1 Cu. Centimeter = .061 + cu. in.
1000 Cu. Centimeters, *cu. cm.* = 1 Cu. Decimeter = 61.026 + " "
1000 Cu. Decimeters, *cu. dm.* = 1 ***Cu. Meter*** = 35.316 + cu. ft.

MEASURES OF CAPACITY

950. The ***Liter*** is the *unit of capacity*, both of liquid and of Dry Measures, and is a vessel whose volume is equal to a cube whose edge is *one-tenth* of a *meter*, equal to 1.05673 Liquid Measure, and .9081 qt. Dry Measure.

TABLE.

10 Milliliters,	*ml.* . . .	= 1 Centiliter.
10 Centiliters,	*cl.* . . .	= 1 Deciliter.
10 Deciliters,	*dl.* . . .	= 1 ***Liter.***
10 LITERS	*L.* . . .	= 1 Dekaliter.
10 Dekaliters,	*Dl.* . . .	= 1 Hectoliter.
10 Hectoliters,	*Hl.* . . .	= 1 Kiloliter, or Stere.
10 Kiloliters,	*Kl.* . . .	= 1 Myrialiter (*Ml.*).

The ***Hectoliter*** is the *unit* in measuring liquids, grain, fruit, and roots in large quantities.

951. EQUIVALENTS IN UNITED STATES MEASURES.

Metric Denominations.	Cubic Measure.	Dry Measure.	Wine Measure.
1 Myrialiter	= 10 Cubic Meters	= 283.72 + bu.	= 2641.4 + gal.
1 Kiloliter	= 1 Cubic Meter	= 28.372 + bu.	= 264.17 gal.
1 Hectoliter	= $\frac{1}{10}$ Cubic Meter	= 2.8372 + bu.	= 26.417 gal.
1 Dekaliter	= 10 Cu. Decimeters	= 9.08 quarts	= 2.6417 gal.
1 ***Liter***	= 1 Cu. Decimeter	= .908 quart	= 1.0567 qt.
1 Deciliter	= $\frac{1}{10}$ Cu. Decimeter	= 6.1022 cu. in.	= .845 gill.
1 Centiliter	= 10 Cu. Centimeters	= .6102 cu. in.	= .338 fluid oz.
1 Milliliter	= 1 Cu. Centimeter	= .061 cu. in.	= .27 fluid dr.

MEASURES OF WEIGHT.

952. The ***Gram*** is the *unit of weight,* and equal to the weight of a cube of distilled water, the edge of which is *one hundredth* of a *meter*, equal to 15.432 Troy grains.

TABLE.

10 Milligrams,	*mg.*	=1 Centigram	=	.15432 +	oz. Troy.
10 Centigrams,	*cg.*	=1 Decigram	=	1.54324 +	" "
10 Decigrams,	*dg.*	=1 ***Gram***	=	15.43248 +	" "
10 GRAMS	*G.*	=1 Dekagram	=	.35273 +	oz. Avoir.
10 Dekagrams,	*Dg.*	=1 Hectogram	=	3.52739 +	" "
10 Hectograms,	*Hg.*	=1 Kilogram, or, ***Kilo***	=	2.20462 +	lb. "
10 Kilograms,	*Kg.*	=1 Myriagram	=	22.04621 +	" "
10 Myriagrams, or, *Mg.* 100 Kilograms		= 1 Quintal	=	220.46212 +	" "
10 Quintals, or *1000* KILOS		=1 Tonneau, or **Ton**	=	2204.62125	" "

The ***Kilogram***, or ***Kilo***, is the *unit* of common weight in trade and is a trifle less than $2\frac{1}{5}$ lb. Avoirdupois.

The ***Tonneau*** is used for weighing very heavy articles, and is about 204 lb. more than a common ton.

953. ***Units*** of the *Common* System may be readily changed to *units* of the *Metric* System by the aid of the following

TABLE.

1 Inch = 2.54 Centimeters.
1 Foot = 30.48 Centimeters.
1 Yard = .9144 Meter.
1 Rod = 5.029 Meters.
1 Mile = 1.6093 Kilometers.
1 Sq. inch = 6.4528 Sq. Centimet.
1 Sq. foot = 929 Sq. Centimeters.
1 Sq. yard = .8361 Sq. Meter.
1 Sq. rod = 25.29 Centiares.
1 Acre = 40.47 Ares.
1 Sq. mile = 259 Hectares.
1 Cu. inch = 16.39 Cu. Centimet.
1 Cu. foot = 28320 Cu. Centimet.
1 Cu. yard = .7646 Cu. Meter.
1 Cord = 3.625 Steres.
1 Fl. ounce = 2.958 Centiliters.
1 Gallon = 3.786 Liters.
1 Bushel = .3524 Hectoliter.
1 Troy gr. = 64.8 Milligrams.
1 Troy lb. = .373 Kilo.
1 Av. lb. = .4536 Kilo.
1 Ton = .907 Tonneau.

954. SYNOPSIS FOR REVIEW.

- MENSURATION.
  - 1. DEFINITION.
  - 2. LINES.
    - 1. Straight Line. 2. Curved Line. 3. Parallel Lines. 4. Horizontal Line. 5. Perpendicular Line.
  - 3. ANGLES.
    - 1. Angle. 2. Right Angle. 3. Obtuse Angle. 4. Acute Angle.
  - 4. PLANE FIGURES.
    - 1. Plane Figure. 2. Polygon. 3. Perimeter. 4. Area. 5. Regular Polygon. 6. Altitude.
  - 5. TRIANGLES.
    - 1. Defs.
      - 1. *Triangle.* 2. *Right-angled Tri.* 3. *Hypothenuse.* 4. *Base.* 5. *Perpendicular.* 6. *Altitude.* 7. *Equilateral Triangle.* 8. *Isosceles Triangle.* 9. *Scalene Triangle.* 10. *Equiangular Triangle.* 11. *Acute-angled Triangle.* 12. *Obtuse-angled Triangle.*
    - 2. Problems. To find … Rule.
      - **882.** *Area of Triangle.*
      - **883.** *Either Dimension.*
      - **884.** *Area of a Triangle.*
      - **886.** *The Hypothenuse.*
      - **887.** *The Base or Perp.*

- MENSURATION—CONTINUED.
  - 6. Q'DRIL'TERALS.
    - 1. Defs.
      - 1. *Quadrilateral.* 2. *Parallelogram.* 3. *Rectangle.* 4. *Square.* 5. *Rhomboid.* 6. *Rhombus.* 7. *Trapezoid.* 8. *Trapezium.* 9. *Altitude.* 10. *Diagonal.*
    - 2. Problems.
      - **898.** **899.** **900.** To find area of
        - *Parallelogram.* Rule.
        - *Trapezoid.* Rule.
        - *Trapezium.* Rule.
  - 7. CIRCLE.
    - 1. Defs. 1. *Circle.* 2. *Diameter.* 3. *Radius.*
    - 2. Problems. To find
      - **904.** *Diam. or Circum.* Rule, 1, 2.
      - **905.** *Area.* Rule, 1, 2.
      - **906.** *Diam. or Circ.* Rule, 1, 2, 3.
      - **907.** *Side of Ins. Sq.* Rule, 1, 2.
      - **908.** *Area of Circular Ring.* Rule.
      - **909.** *Mean Proportional.* Rule.
  - 8. SIMILAR PLANE FIGURES.
    - 1. Definition.
    - 2. Principles, 1, 2, 3, 4, 5.
  - 9. SOLIDS.
    - 1. Defs.
      - 1. *Solid or Body.* 2. *Prism.* 3. *Altitude.* 4. *Parallelopipedon.* 5. *Cube.* 6. *Cylinder.* 7. *Pyramid.* 8. *Cone.* 9. *Altitude of Pyramid or Cone.* 10. *Slant Height.* 11. *Frustum.* 12. *Sphere.* 13. *Diameter.* 14. *Radius.*
    - 2. Problems. To find
      - **918.** *Conv. Surf. of Prism or Cyl.* Rule.
      - **919.** *Volume " " "* Rule.
      - **925.** *Conv. Surf. of Pyr. or Cone.* Rule.
      - **926.** *Volume " " " "* Rule.
      - **927.** *Conv Surf. of Frustum.* Rule.
      - **928.** *Volume " "* Rule.
      - **932.** *Surface of Sphere.* Rule.
      - **933.** *Volume " "* Rule.
      - **934.** *Dim. of Rectang. Solid.* Rule.
    - 3. Similar Solids.
      - 1. *Definitions.*
      - 2. *Principles,* 1, 2.
  - 10. GAUGING. 1. Definitions. 2. Rules.
- METRIC SYSTEM.
  - 1. DEFINITIONS.
    - 1. Metric System. 2. Meter. 3. Multiple Units. 4. Sub-Multiple Units.
  - 2. MEASURES OF EXTENSION.
    - 1. Meter. 2. Kilometer. 3. Are. 4. Square Meter. 5. Stere. 6. Cubic Meter.
  - 3. MEASURES OF CAPACITY.
    - 1. Liter. 2. Hectoliter. Equiv. in U. S. Measures.
  - 4. MEASURES OF WEIGHT.
    - 1. Gram. 2. Kilogram. 3. Tonneau. Equivalent Units.

APPENDIX

MEASURES

1. A ***Measure*** is a *standard unit,* established by law or custom, by which *quantity,* as extent, dimension, capacity, amount, or value is measured or estimated.

Thus, the *standard unit* of Measures of Extension is the *yard;* of Liquid Measure, the *wine gallon;* of Dry Measure, the *Winchester bushel;* of Weight, the *Troy pound,* etc. Hence the length of a piece of cloth is ascertained by applying the *yard* measure; the capacity of a cask, by the use of the *gallon* measure; of a bin, by the use of the *bushel* measure; the weight of a body, by the *pound* weight, etc.

2. Measures may be classified into *six kinds*:

1. Extension.
2. Capacity.
3. Weight.
4. Time.
5. Angles or Arcs.
6. Money or Value.

MEASURES OF EXTENSION.

3. ***Extension*** has *length, breadth,* and *thickness.*

4. A ***Line*** has *length* only.

5. A ***Surface or Area*** has *length* and *breadth.*

6. A ***Solid*** has *length, breadth,* and *thickness.*

LINEAR MEASURE.

7. ***Linear Measure,*** also called ***Long Measure*** is used in measuring lines and distances.

TABLE.

12	Inches (*in.*)	= 1 Foot	*ft.*	1 *Mi.* =	63360 *in.*
3	Feet	= 1 Yard	*yd.*		5280 *ft.*
5½ 16½	Yards, or Feet	= 1 Rod	*rd.*		1760 *yd.*
320	Rods	= 1 Mile	*mi.*		320 *rd.*

8. ***Cloth Measure*** is practically out of use. In measuring goods sold by the yard, the yard is divided into *halves, fourths, eighths,* and *sixteenths.*

2¼	Inches	= 1 Sixteenth,	$\frac{1}{16}$ *yd.*
2	Sixteenths, (4½ in.)	= 1 Eighth,	⅛ *yd.*
2	Eighths, (9 in.)	= 1 Quarter,	¼ *yd.*
4	Quarters	= 1 Yard,	1 *yd.*

At U. S. Custom-Houses, in estimating duties, the yard is divided into *tenths* and *hundredths.*

9. *Mariners* use the following denominations :

9	Inches	= 1 Span,	*Sp.*
8	Spans or 6 Ft.	= 1 Fathom,	*fath.*
120	Fathoms	= 1 Cable's Length,	*c. l.*
7⅓	C. Length	= 1 Nautical Mile (or Knot),	*mi.*
3	Miles	= 1 League,	*lea.*

10. In *geographical* and *astronomical* calculations :

1 Geographic Mile	= 1.15 Statute Miles.	
3 " "	= 1 League.	
60 " " or *69.16* Statute "	= 1 Deg.	of Latitude on a Meridian, or of Long. on the Equator.
360 Degrees	= the Circumference of the Earth.	

11. The following are sometimes used:

3 Barley-corns, or Sizes	= 1 Inch.	Used by shoemakers.
4 Inches	= 1 Hand.	" to measure the height of horses at the shoulder.
$3\frac{3}{10}$ Feet	= 1 Pace.	
3 Inches	= 1 Palm.	
21.888 Inches	= 1 Sacred Cubit.	
6 Points	= 1 Line.	Used in clock-making.
12 Lines	= 1 Inch.	

1. The nautical mile (or knot) is the same as the geographical mile, and is used in measuring the speed of vessels.

2. The geographical mile is $\frac{1}{60}$ of $\frac{1}{360}$ or $\frac{1}{21600}$ of the distance round the center of the earth. It is a small fraction more than 1.15 statute miles.

3. The length of a degree of latitude varies, being 68.72 miles at the equator, 68.9 to 69.05 miles in middle latitudes, and 69.30 to 69.34 miles in the polar regions. The mean or average length, 69.16, is the standard recently adopted by the U. S. Coast Survey. A degree of longitude is greatest at the equator, where it is 69.16 miles, and it gradually decreases toward the poles, where it is 0.

12. ***Surveyors' Linear Measure*** is used by land surveyors in measuring roads and boundaries of land.

TABLE.

7.92 Inches = 1 Link	*l.*	1 *Mi.* =	63360 *in.*
25 Links = 1 Rod	*rd.*		8000 *l.*
4 Rods = 1 Chain	*ch.*		320 *rd.*
80 Chains = 1 Mile	*mi.*		80 *ch.*

1. A ***Gunter's Chain*** is the *unit* of measure, and is 4 rods, or 66 feet long, and consists of 100 links.

2. *Engineers* commonly use a chain or *measuring tape*, 100 feet long, each foot divided into *tenths*.

3. *Measurements* are recorded in *chains* and hundredths.

13. COMPARISON OF DISTANCES.

Country.	Distance.	U. S. mile.	Country.	Distance.	U. S. mile.
England,	1 Mile	= 1	Russia,	1 Verst	= .66
France,	1 Km.	= .62	Turkey,	1 Berri	= 1.04
Spain,	1 League	= 4.15	Portugal,	1 Milha	= 1.28
Prussia,	1 Meile	= 4.93	Persia,	1 Farsang	= 4.17
Austria,	1 Meile	= 4.98	China,	1 Li	= .35
Sweden,	1 Mil	= 6.64	Egypt,	1 Mili	= 1.15
Switzerland,	1 Lieue	= 2.98	East Indies,	1 Coss	= 1.14
Mexico,	1 Silio	= 6.76	Japan.	1 Ri	=2.562

SURFACE OR SQUARE MEASURE.

14. ***Surface or Square Measure*** is used in computing areas or surfaces; as of land, boards, painting, plastering, paving, etc.

TABLE.

144 Square Inches (*sq. in.*) = 1 Square Foot . . *sq. ft.*
9 Square Feet = 1 Square Yard . . *sq. yd.*
30¼ Square Yards = 1 Sq. Rod or Perch *sq. rd.; P.*
160 Square Rods = 1 Acre . . . . . . *A.*
640 Acres = 1 Square Mile . . *sq. mi.*

sq. mi.	*A.*	*sq. rd.*	*sq. yd.*	*sq. ft.*	*sq. in.*
1 =	640 =	102400 =	3097600 =	27878400 =	4014489600
	1 =	160 =	4840 =	43560 =	6272640
		1 =	30¼=	272¼=	39204
			1 =	9 =	1296
				1 =	144

15. ***Artificers*** estimate their work as follows:

By the *square foot;* as in glazing, stone-cutting, etc.

By the *square yard*, or by the *square* of 100 square feet; as in plastering, flooring, roofing, paving, etc.

One thousand shingles, averaging 4 in. wide, and laid 5 in. to the *weather*, are estimated to be a *square*.

16. *Surveyors' Square Measure* is used by surveyors in computing the area or contents of *land.*

TABLE.

625 Square Links (*sq. l.*) = 1 Pole . . . . . . *P.*
16 Poles = 1 Square Chain . . *sq. ch.*
10 Square Chains = 1 Acre . . . . . *A.*
640 Acres = 1 Square Mile . . . *sq. mi.*
36 Square Miles (6 miles square) = 1 Township . . . *Tp.*

Tp.	*sq. mi.*	*A.*	*sq. ch.*	*P.*	*sq. l.*
1 =	36 =	23040 =	230400 =	3686400 =	2304000000
	1 =	640 =	6400 =	102400 =	6400000
		1 =	10 =	100 =	100000

1. The ***Acre*** is the *unit* of land measure.

2. Government lands are divided into *Townships*, by parallels and meridians, each containing 36 square miles or *Sections*. Each section contains 640 acres (1 sq. mile), and is subdivided into *half-sections, quarter-sections*, etc.

3. Measurements of land are commonly recorded in *square miles, acres*, and *hundredths* of an acre. The *rood* is no longer used.

CUBIC OR SOLID MEASURE.

17. *Cubic* or ***Solid Measure*** is used in computing the contents of solids; as timber, wood, stone, boxes of goods, the capacity of rooms, etc.

TABLE.

1728 Cubic In. (*cu. in.*) = 1 Cubic Ft., *cu. ft.*
27 Cubic Ft. = 1 Cubic Yd., *cu. yd.*

1 *cu. yd.* = 46656 *cu. in.*
1 *cu. yd.* = 27 *cu. ft.*

1. A *Register Ton*, used in measuring the entire internal capacity or *tonnage* of vessels, is 100 cubic feet.

2. A *Shipping Ton*, used in measuring *cargoes*, is 40 cubic feet in the U. S., and in England 42 cubic feet.

18. *Wood Measure* is used to measure wood and rough stone.

TABLE.

16	Cubic Feet	= 1 Cord Foot	*cd. ft.*
8 128	Cord Feet, or Cubic Feet	= 1 Cord	*Cd.*
24¾	Cubic Feet	= 1 Perch of Stone, or of Masonry.	*Pch.*

A *Cord* of wood is a pile 8 ft. long, 4 ft. wide, and 4 ft. high.

A *cord-foot* is 1 ft. in length of such a pile; that is, 1 ft. long, 4 ft. wide, and 4 ft. high.

A *Perch* of stone or of masonry is 16½ ft. long, 1½ ft. wide, and 1 ft. high. Stone-masons usually call 25 cu. ft. a perch.

19. *Duodecimals* are the parts of a unit resulting from continually dividing by 12, and are sometimes used in measuring surfaces and solids.

TABLE.

12 Fourths ('''')	= 1 Third . . .	1'''	1 *Ft.* =	20736''''
12 Thirds	= 1 Second . . .	1''		1728'''
12 Seconds	= 1 Prime . .	1'		144''
12 Primes	= 1 Foot	*Ft.*		12'

The marks ', '', ''', '''', are called indices.

Railroad and transportation companies estimate light freight by the space it occupies in *cubic feet*, but heavy freight by *weight*.

Masonry is estimated by the *cubic foot*, and *perch;* also by the *square foot* and *square yard.*

Materials are usually estimated by cubic measure; the *work* by cubic, or by square measure.

Engineers, in making estimates for *excavations* and *embankments*, take the dimensions with a line or measure divided into feet and decimals of a foot. The computations are made in feet and decimals, and the results are reduced to *cubic yards.* In civil engineering, the *cubic yard* is the *unit* to which estimates for excavations and embankments are finally reduced.

A cubic yard of common earth is called a *load*.

Brickwork is generally estimated by the 1000 *bricks*, sometimes in *cubic feet*.

Bricks are of various dimensions. The average size of a *common* brick is 8 in. long, 4 in. wide, and 2 in. thick.

Philadelphia or Baltimore front bricks are $8\frac{1}{4} \times 4\frac{1}{8} \times 2\frac{3}{8}$ inches; North River bricks, $8 \times 3\frac{1}{2} \times 2\frac{1}{4}$ inches; Maine bricks, $7\frac{1}{2} \times 3\frac{3}{8} \times 2\frac{3}{8}$; and Milwaukee bricks, $8\frac{1}{2} \times 4\frac{1}{8} \times 2\frac{3}{8}$ inches.

A cubic foot is estimated to contain 27 bricks laid dry. Laid in mortar, an allowance is made of from $\frac{1}{10}$ to $\frac{1}{8}$ for the mortar.

Five courses of bricks in the height of a wall are called a foot.

A brick wall which is a brick and a half thick is said to be of the *standard thickness*.

In estimating *material*, allowance is made for doors, windows, and cornices.

In estimating the *work*, masons measure each wall on the *outside*. Ordinarily, no allowance is made for doors, windows, and cornices, but sometimes an allowance of *one-half* is made, this being a matter of contract.

In scaling or measuring timber for shipping or freighting, $\frac{1}{5}$ of the solid contents of round timber is deducted for waste in hewing or sawing. Thus, a log that will make 40 feet of hewn or sawed timber, actually contains 50 cubic feet by measurement; but its market value is only equal to 40 cubic feet of hewn or sawed timber.

Sawed timber, joists, plank, and scantlings are generally bought and sold by what is called *board measure*. *Hewn* and *round* timber by *cubic measure*.

In board and lumber measure, all estimates are made on 1 inch in thickness; in buying and selling lumber, *one-fourth* the price is added for every $\frac{1}{4}$ inch thickness over an inch.

In *Board Measure* all boards are assumed to be 1 in. thick.

A *board foot* is 1 ft. long, 1 ft. wide, and 1 inch thick; hence 12 board feet make 1 *cubic foot*.

Board feet are changed to cubic feet by dividing by 12.

Cubic feet are changed to board feet by multiplying by 12.

MEASURES OF CAPACITY.

20. ***Capacity*** signifies extent of *room* or *space*.

21. Measures of capacity are divided into two classes; Measures of Liquids and Measures of Dry Substances.

22. The ***Units*** of Capacity are the *Gallon* for Liquid, and the *Bushel* for Dry Measure.

LIQUID MEASURE.

23. ***Liquid Measure*** is used in measuring liquids; as spirituous liquors, oil, molasses, milk, water, etc.

TABLE.

4 Gills (*gi.*) = 1 Pint . . . *pt.*
2 Pints = 1 Quart . . . *qt.*
4 Quarts = 1 Gallon . . . *gal.*

$$1\ Gal. = \begin{cases} 32\ gi. \\ 8\ pt. \\ 4\ qt. \end{cases}$$

The *Standard Liquid Gallon* of the United States contains 231 cubic inches, and is equal to about 8⅓ lb. Avoir. of pure water.

The *Imperial Gallon* of Great Britain contains 277.274 cubic inches, and is equal to about 1.2 U. S. liquid gallons.

The *Old Ale or Beer Measure* is out of use. The gallon contained 282 cubic inches.

24. In estimating the capacity of cisterns, reservoirs, etc.:

31½ Gallons make 1 Barrel . . . *bbl.*
63 Gallons " 1 Hogshead . . *hhd.*

1. The barrel and hogshead are not fixed measures, but vary when used for commercial purposes, the former containing from 28 to 45 gallons, the latter from 60 to 125 gallons.

2. In some of the New England States the barrel is estimated at 32 gallons; in some States 31½ gallons, and in others from 28 to 32.

3. The tierce, hogshead, pipe, puncheon, butt and tun are the name of casks, and do not express any fixed or definite measures. They are usually gauged, and have their capacities in gallons marked on them.

25. *Apothecaries' Fluid Measure* is used by physicians and apothecaries in prescribing and compounding *liquid* medicines.

TABLE.

60 Minims, or drops (♏ or *gtt.*)	=	1 Fluidrachm	*f*ʒ.
8 Fluidrachms	=	1 Fluidounce	*f*℥.
16 Fluidounces	=	1 Pint	*O.*
8 Pints	=	1 Gallon	*Cong.*

$$Cong.\ 1 = O.\ 8 = f℥\ 128 = fʒ\ 1024 = ♏\ 61440.$$

O. is an abbreviation of *octans,* the Latin for *one-eighth*; *Cong.* for *congiarium,* the Latin for gallon.

A common teaspoonful, or 45 drops, makes about one fluidrachm. A common teacup holds about 4 fluidounces; a common tablespoon, about half a fluidounce; a pint of water weighs a pound.

℞ is an abbreviation for *recipe,* or take; ā., aa., for equal quantities; j. for 1; ij. for 2; *ss.* for *semi,* or half; gr. for grain; P. for a little part; P. æq. for equal parts; q. p., as much as you please.

26. COMPARISON OF LIQUID MEASURES.

Country.	Measure.		U. S. gal.	Country.	Measure.		U. S. gal.
England,	1 Gal.	=	1.2	Switzerland,	1 pot	=	.40
France,	1 Dl.	=	2.64	Turkey,	Almud	=	1.38
Prussia,	1 Quart	=	.30	Mexico,	1 Fasco	=	.63
Austria,	1 Maas	=	.37	•Brazil,	1 Medida	=	.74
Sweden,	1 Kanna	=	.69	Cuba,	1 Arroba	=	4.01
Denmark,	1 Kande	=	.51	South Spain,	1 Arroba	=	4.25

DRY MEASURE.

27. *Dry Measure* is used in measuring articles not liquid; as grain, fruit, salt, roots, etc.

TABLE.

2 Pints (*pt.*)	=	1 Quart	*qt.*	1 *Bu.* =	64 *pt.* 32 qt. 4 pk.
8 Quarts	=	1 Peck	*pk.*		
4 Pecks	=	1 Bushel	*bu.*		

The *Standard Bushel* of the United States contains 2150.42 cubic inches, and is a cylindrical measure 18½ inches in diameter and 8 inches deep.

The half-peck, or *dry gallon*, contains 268.8 cubic inches. *Six* quarts dry measure are equal, to nearly 7 quarts, liquid measure.

The *Imperial Bushel* of Great Britain contains 2218.192 cu. in.

The *English Quarter* contains 8 Imperial bushels, or 8¼ U. S. bushels.

Grain is shipped from New York by the *Quarter* of 480 lb. (8 U. S. bu.), or by the *Ton* of 33⅓ U. S. bushels.

The bushel, *heap measure*, is the Winchester bushel, heaped in the form of a cone, not less than 6 inches high and 19½ inches in diameter, equal to the outside diameter of the standard bushel measure, and equal to 2747.715 cu. in.

Grain, seeds, and small fruits are sold by *stricken measure*, or the measure must be *even* full.

Corn in the ear, potatoes, coal, large fruits, coarse vegetables and other bulky articles, are sold by *heap measure.*

It is sufficiently accurate in practice to call 5 *stricken* measures equal to 4 *heaped* measures.

The value of many kinds of grain, seeds, fruit, and other articles, are often determined by weight instead of by bulk.

American coal is bought and sold, in large quantities, by the *ton;* in small quantities, by the *bushel.*

The liquid and dry measures of the same denomination are of different capacities. The *exact* and the *relative* size of each may be readily seen by the following

28. COMPARATIVE TABLE OF MEASURES OF CAPACITY.

	Cubic in. in one gallon.	Cubic in. in one quart.	Cubic in. in one pint.	Cubic in. in. one gill.
Liquid measure	231	57¾	28⅞	$7\frac{7}{32}$
Dry measure (½ pk.) . .	268⅘	67⅕	33⅗	8⅖

A cubic foot of pure water weighs 1000 oz., 62½ lb. Avoir.

29. COMPARISON OF GRAIN MEASURES.

Country.	Measure.	U. S. bush.	Country.	Measure.	U. S. bush.
England,	1 Bushel	= .973	Germany,	1 Schef.	= 1.5 to 3
France,	1 Hectoliter	= 2.84	Persia,	1 Artaba	= 1.85
Prussia,	1 Scheffel	= 1.56	Turkey,	1 Kiló	= 1.03
Austria,	1 Metze	= 1.75	Brazil,	1 Fan.	= 1.5
Russia,	1 Chetverik	= .74	Mexico,	1 Alque.	= 1.13
Greece,	1 Kailon	= 2.837	Madras,	1 Parah	= 1.743

ENGLISH MEASURES OF CAPACITY.

30. *Wine Measure* is used to measure wines and all liquids, except malt liquors and water.

TABLE.

4 Gills = 1 Pint . . . . *pt.*
2 Pints = 1 Quart . . . . *qt.*
4 Quarts = 1 Gallon . . . *gal.*
10 Gallons = 1 Anker . . . *ank.*
18 Gallons = 1 Runlet . . . *run.*
42 Gallons = 1 Tierce . . . . *tier.*
2 Tierces = 1 Puncheon . . *pun.*
63 Gallons = 1 Hogshead . . *hhd.*
2 Hogsheads = 1 Pipe . . . . *pipe.*
2 Pipes = 1 Tun . . . . *tun.*

31. *Ale and Beer Measure* is used to measure all malt liquors and water.

TABLE.

2 Pints = 1 Quart . . . . *qt.*
4 Quarts = 1 Gallon . . . *gal.*
9 Gallons = 1 Firkin. . . . *fir.*
18 Gallons = 1 Kilderkin . . *kil.*
36 Gallons = 1 Barrel . . . *bar.*
1½ Bar. or 54 gal. = 1 Hogshead . . *hhd.*
2 Hogsheads = 1 Butt . . . . butt.
2 Butts = 1 Tun. . . . . tun.

32. *Corn or Dry Measure* is used to measure all dry commodities not usually *heaped.*

TABLE.

2 Quarts	= 1 Pottle	*pot.*
2 Pottles	= 1 Gallon	*gal.*
2 Gallons	= 1 Peck	*pk.*
4 Pecks	= 1 Bushel . . .	*bus.*
2 Bushels	= 1 Strike	*str.*
4 Bushels	= 1 Coomb	*coomb.*
2 Coombs or 8 bu.	= 1 Quarter . . .	*qr.*
5 Quarters	= 1 Load	*load.*
2 Loads or 10 Qr.	= 1 Last	*last.*

14 Pounds = 1 Stone.
21½ Stones = 1 Pig of iron or lead.
8 Pigs = 1 Fother.

The *stone* varies. Legally it is 14 lb. A stone of butcher's meat or fish is reckoned at 8 lb.; of cheese, at 16 lb.; of hemp, at 32 lb.

A *pig* of iron or lead is 250 lb., and 8 pigs make a *fother.*

MEASURES OF WEIGHT.

33. *Weight* is the measure of the quantity of matter a body contains, determined by the force of gravity.

34. The ***Standard Unit*** of weight is the *Troy Pound* of the Mint, and contains 5760 grains.

TROY WEIGHT.

35. ***Troy Weight*** is used in weighing gold, silver, jewels, and in philosophical experiments.

TABLE.

24 Grains (*gr.*)	= 1 Pennyweight .	*pwt.*		5760 *gr.*
20 Pennyweights	= 1 Ounce	*oz.*	1 *lb.* =	240 *pwt.*
12 Ounces	= 1 Pound	*lb.*		12 *oz.*

36. TABLE.

DIAMOND WEIGHT.

16 Parts = 1 Grain.
4 Grains = 1 Carat.
1 Carat = 3⅕ Troy gr., nearly.

ASSAYERS' WEIGHT.

1 Carat =10 pwts.
1 Carat gr. = 2 pwts. 12 gr. or 60 Troy gr.
24 Carats = 1 Troy lb.

The term *carat* is also used to express the *fineness* of gold, each carat meaning a *twenty-fourth* part.

APOTHECARIES' WEIGHT.

37. ***Apothecaries' Weight*** is used by apothecaries and physicians in compounding dry medicines.

TABLE.

20 Grains (*gr.* xx) = 1 Scruple . . . . *sc.*, or ℈.
3 Scruples (℈ iij) = 1 Dram . . . . . *dr.*, or ʒ.
8 Drams (ʒ viij) = 1 Ounce . . . . . *oz.*, or ℥.
12 Ounces (℥ xij) = 1 Pound . . . . . *lb.*, or ℔.

℔ 1 = ℥ 12 = ʒ 96 = ℈ 288 = *gr.* 5760.

1. *Medicines* are bought and sold in quantities by Avoirdupois weight.

2. The pound, ounce and grain are the same as those of Troy weight, the ounce being *differently divided.*

AVOIRDUPOIS WEIGHT.

38. ***Avoirdupois Weight*** is used for all the ordinary purposes of weighing.

TABLE.

16 Ounces (*oz.*) = 1 Pound . . . *lb.*
100 Pounds = 1 Hundredweight *cwt.*
20 cwt., or 2000 lb. = 1 Ton . . . . *T.*

$$1\ T. = \begin{cases} 32000\ oz. \\ 2000\ lb. \\ 20\ cwt. \end{cases}$$

The *ounce is often* divided into *halves*, *quarters*, etc.

39. The *Long* or *Gross ton, hundred-weight*, and *quarter* were formerly in common use; but they are now seldom used except in estimating duties at the United States Custom-Houses, in freighting and wholesaling coal from the Pennsylvania mines.

LONG TON TABLE.

28 Pounds	= 1 Quarter	*qr.*	1 *T.* =	2240 *lb.*
4 Qr., or 112 lb.	= 1 Hundredweight	*cwt.*		80 *qr.*
20 cwt. or 2240 lb.	= 1 Ton	*T.*		20 *cwt.*

40. The following denominations are also used:

100 Pounds of Grain or Flour	make	1 Cental.
100 Pounds of Dry Fish	"	1 Quintal.
100 Pounds of Nails	"	1 Keg.
196 Pounds of Flour	"	1 Barrel.
200 Pounds of Pork or Beef	"	1 Barrel.
280 Pounds	"	1 Barrel of Salt at the N. Y. Salt Works.

41. COMPARATIVE TABLE OF WEIGHTS.

	TROY.	APOTHECARIES.	AVOIRDUPOIS.
1 Pound	= 5760 Grains	= 5760 Grains	= 7000 Grains.
1 Ounce	= 480 "	= 480 "	= 437.5 "
	175 Pounds	= 175 Pounds	= 144 Pounds.

42. COMPARISON OF COMMERCIAL WEIGHTS.

Country.	Weight.	U. S. lbs. avdp.	Country.	Weight..	U. S. lbs. avdp.
France,	1 Kilogram	= 2.20	Prussia,	1 Zolpf'd	= 1.10
Germany,	1 Pfund	= 1.10	Netherlands,	1 Pond	= 2.20
Austria,	1 Pfund	= 1.23	East Indies,	1 Seer	= 2.06
Russia,	1 Funt	= .90	China,	1 Catty	= 1.33
Sweden,	1 Pund	= .93	Japan,	1 Kin	= .62
Denmark,	1 Pund	= 1.10	Mexico,	1 Libra	= 1.02
Turkey,	1 Oka	= 2.82	Brazil,	1 Arratel	= 1.02
Egypt,	1 Rottoli,	= 1.008	Spain,	1 Libra	=1.016
Persia,	1 Rattel,	= 2.116	Sicily,	1 Libra	= .7
Madras,	1 Vis	= 3.125	Arabia,	1 Maund	= 3

43. The weight of the *bushel* of certain grains, seeds and vegetables has been fixed by statute in many of the States; and these statute weights must govern in buying and selling, unless specific agreements to the contrary be made.

TABLE OF AVOIRDUPOIS POUNDS IN A BUSHEL,

As prescribed by statute in the several States named.

COMMODITIES.	*California.*	*Connecticut.*	*Illinois.*	*Indiana.*	*Iowa.*	*Kentucky.*	*Louisiana.*	*Maine.*	*Massachusetts.*	*Michigan.*	*Minnesota.*	*Missouri.*	*New Jersey.*	*New York.*	*Ohio.*	*Oregon.*	*Pennsylvania.*	*Rhode Island.*	*Vermont.*	*Washington T.*	*Wisconsin.*
Barley	50		48	48	48	48	32		46	48	48	48	48	48	48	46	47		46	45	48
Beans			60	60	60	60						60		62							
Blue Grass Seed			14	14	14	14						14									
Buckwheat	40	45	40	50	52	52			46	42	42	52	50	48		42	48		46	42	42
Castor Beans			46	46	46							46									
Clover Seed			60	60	60	60				60	60	60	64	60	60	60				60	60
Dried Apples			24	25	24					28	28	24				28				28	28
Dried Peaches			33	33	33					28	28	33				28				28	28
Flax Seed			56	56	56	56						56	55	55	56						56
Hemp Seed			44	44	44	44						44									
Indian Corn	52	56	52	56	56	56	56		56	56	56	52	56	58	56	56	56		56	56	56
Indian Corn in ear			70	68	68																
Indian Corn Meal			48	50		50		50	50									50			
Oats	32	28	32	32	35	33⅓	32	30	30	32	32	35	30	[illegible]	[illegible]	34	32		32	36	32
Onions			57	48	57	57			52			57						50		50	
Potatoes		60	60	60	60	60		60				60	60	60		60		60	60	60	60
Rye	54	56	54	56	56	56	32		56	56	56	56	56	56	56	56	56		56	56	56
Rye Meal								50	50									50			
Salt				50	50	50						50		56							
Timothy Seed			45	45	45	45						45		44							46
Wheat	60	56	60	60	60	60	60		60	60	60	60	60	60	60	60	60		60	60	60
Wheat Bran			20		20	20						20									

In Pennsylvania 80 lbs. coarse, 70 lbs. ground, or 62 lbs. fine salt make 1 bushel; and in Illinois, 50 lbs. common or 55 lbs. fine salt make 1 bushel.

In Maine 64 lbs. of ruta baga turnips or beets make 1 bushel.

A cask of lime is 240 lbs. Lime in slaking absorbs 2½ times its *volume, and 2¼ times its* weight in water.

44. The following table will assist farmers in making an accurate estimate of the amount of land in different fields under cultivation.

TABLE.

10 rods × 16 rods = 1 A.
8 " × 20 " = 1 "
5 " × 32 " = 1 "
4 " × 40 " = 1 "
5 yds. × 968 yds. = 1 "
10 " × 484 " = 1 "
20 " × 242 " = 1 "
40 " × 121 " = 1 "

220 feet × 198 feet = 1 A.
110 " × 369 " = 1 "
60 " × 726 " = 1 "
120 " × 363 " = 1 "
200 " × 108.9 " = ½ "
100 " × 145.2 " = ⅓ "
100 " × 108.9 " = ¼ "

45. The following table will often be found convenient, taking *inside* dimensions:

A box 24 in. × 24 in. × 14.7 will contain *a barrel* of 31½ gallons.
A box 15 in. × 14 in. × 11 in. will contain 10 *gallons.*
A box 8¼ in. × 7 in. × 4 in. will contain *a gallon.*
A box 4 in. × 4 in. × 3.6 in. will contain *a quart.*
A box 24 in. × 28 in. × 16 in. will contain 5 *bushels.*
A box 16 in. × 12 in. × 11.2 in. will contain *a bushel.*
A box 12 in. × 11.2 in. × 8 in. will contain *a half-bushel.*
A box 7 in. × 6.4 in. × 12 in. will contain *a peck.*
A box 8.4 in. × 8 in. × 4 in. will contain *a half-peck* or 4 dry quarts.
A box 6 in. by 5⅜ in., and 4 in. deep, will contain *a half-gallon.*
A box 4 in. by 4 in. and 2⅒ in. deep, will contain *a pint.*

46. Nails are put up 100 pounds to the keg.

SIZE.	Length, inches.	Nails in a lb.	SIZE.	Length, inches.	Nails in a lb.	SIZE.	Length, inches.	Nails in a lb.
3*d* fine blued.	1⅛	725	30*d* com. blued.	4½	16	6*d* casing	2	210
3*d* com. "	1¼	400	40*d* " "	5	14	8*d* "	2½	134
4*d* " "	1½	300	50*d* " "	5½	11	10*d* "	3	78
6*d* " "	2	150	60*d* " "	6	8	6*d* finishing	2	317
8*d* " "	2½	85	6*d* fence.	2	80	8*d* "	2½	208
10*d* " "	3	60	8*d* "	2½	50	10*d* "	3	126
12*d* " "	3¼	50	10*d* "	3	30	6*d* clinching	2	118
16*d* " "	3½	40	12*d* "	3¼	27	8*d* "	2½	80
20*d* " "	4	20	16*d* "	3½	20	10*d* "	3	45

5 lbs. of 4*d* or 3¾ lbs. of 3*d* will put on 1,000 shingles.
5¾ lbs. of 3d fine will put on 1,000 lath.

MEASURES OF TIME.

47. ***Time*** is the measure of duration.

48. The ***Unit*** is the *mean solar day* .

TABLE.

60 Seconds (*sec.*)	= 1 Minute	*min.*
60 Minutes	= 1 Hour	*hr.*
24 Hours	= 1 Day	*da.*
7 Days	= 1 Week	*wk.*
365 Days, or 12 Calendar Mo.	= 1 Common Year	*yr.*
366 Days	= 1 Leap Year	*yr.*
100 Years	= 1 Century	*Cen.*

COMMON YEAR.

1 *Yr.* =	525600 *min.*
	8760 *hr.*
	12 *mo.*
	365 *da.*

1. Every year that is exactly divisible by 4 is a *leap* year, the centennial years excepted ; the other years are *common* years.

2. Every centennial year that is divisible by 400 is a leap year.

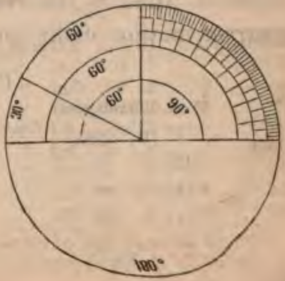
3. In most business transactions 30 days are called 1 month, and 12 months 1 year.

4. The *civil day* begins and ends at 12 o'clock, midnight. A. M. denotes the time before noon ; M., at noon ; and P. M., afternoon.

5. The *astronomical day*, used by astronomers in dating events, begins and ends at 12 o'clock, noon.

MEASURE OF ANGLES.

49. ***Circular*** or ***Angular Measure*** is used in measuring angles and arcs of circles, in determining latitude and longitude, the location of places, the motion of the heavenly bodies, etc.



50. The ***Unit*** is the *degree,* which is $\frac{1}{360}$ part of the circumference of any circle.

TABLE.

60 Seconds (″) = 1 Minute	′	1 *C.* =	1296000″.
60 Minutes = 1 Degree	°		21600′
30 Degrees = 1 Sign	*S.*		360°
12 Signs, or 360° = 1 Circle	*C.*		12 *S.*

A ***Semi-Circumf'ce*** is *one-half* of a circumference, or 180°.

A ***Quadrant*** is *one-fourth* of a circumference, or 90°.

A ***Sextant*** is *one-sixth* of a circumference, or 60°.

A ***Sign*** is *one-twelfth* of a circumference, or 30°.

A ***Degree*** (1°) is *one-ninetieth* of a *right angle.*

The length of a degree varies with the size of the circle; thus, a degree of longitude at the Equator is 69.16 statute miles, at 30° of latitude it is 59.81 miles, at 60° of latitude it is 34.53 miles, and at 90°, or the poles, it is nothing.

A *minute* of the earth's circumference is called a geographic mile.

LONGITUDE AND TIME.

51. Since the earth performs one complete revolution on its axis in a day or 24 hours, the sun *appears to pass* from east to west round the earth, or through 360° of *longitude,* once in every 24 hours of *time.* Hence the relation of time to the *real* motion of the earth or the *apparent* motion of the sun, is as follows:

TABLE.

For a difference of	There is a difference of
15° in Long.	1 hr. in Time.
15′ " "	1 min. " "
15″ " "	1 sec. " "
1° " "	4 min. " "
1′ " "	4 sec. " "
1″ " "	$\frac{1}{15}$ sec. " "

COUNTING.

52. This measure is used in counting certain classes of articles for market purposes.

TABLE.

12 Units	= 1 Dozen	*doz.*	1 *G. gro.* =	1728 *units.*
12 Dozen	= 1 Gross	*gro.*		144 *doz.*
12 Gross	= 1 Great Gross	*G. gro.*		12 *gro.*
20 Units	= 1 Score	*sc.*		

Two things of a kind are often called a *pair*, and *six* things a *set*; as a *pair* of horses, a *set* of chairs, etc.

PAPER.

53. The denominations of this table are used in the paper trade

TABLE.

24 Sheets	= 1 Quire	*qr.*	1 *B.* =	4800 Sheets.
20 Quires	= 1 Ream	*rm.*		200 Quires.
2 Reams	= 1 Bundle	*bun.*		10 Reams.
5 Bundles	= 1 Bale	*B.*		5 Bundles.

Paper is bought at *wholesale* by the bale, bundle, and ream; and at *retail* by the ream, quire, and sheet.

Paper may be made to order of any size, but the greater part made up for sale is only of regular sizes. The names generally define the sizes. Writing and Drawing Papers differ in size from Printing Papers of the same name. English sizes differ from American. However, when English or French printing papers are made for this country they are of American sizes.

54. SIZE OF WRITING PAPERS.

FOLDED PAPERS.

	Inches.		Inches.
Billet Note	6 × 8	Letter	10 × 16
Octavo Note	7 × 9	Commercial Letter	11 × 17
Commercial Note	8 × 10	Packet Post	11½ × 18
Packet Note	9 × 11	Extra Packet Post	11½ × 18½
Bath Note	8½ × 14	Foolscap	12½ × 16

The dimensions given above are those in most general use. Some kinds occasionally vary a trifle.

55. FLAT PAPERS.

	Inches.		Inches.
Law Blank	13 × 16	Medium	18 × 23
Flat Cap	14 × 17	Royal	19 × 24
Crown	15 × 19	Super Royal	20 × 28
Demy	16 × 21	Imperial	22 × 30
Folio Post	17 × 22	Elephant	22¼ × 27¾
Check Folio	17 × 24	Columbia	23 × 33¼
Double Cap	17 × 28	Atlas	26 × 33
Extra Size Folio	19 × 23	Double Elephant	26 × 40

Extra Size Folio is sometimes 18 × 23 inches, and 19 × 24 inches. Imperial is sometimes 93 × 31 inches.

56. SIZE OF PRINTING PAPERS.

	Inches.		Inches.
Medium	19 × 24	Double Medium	24 × 38
Royal	20 × 25	Double Royal	26 × 40
Super Royal	22 × 28	Double Super Royal	28 × 42
Imperial	22 × 32	" " "	29 × 43
Medium-and-half	24 × 30	Broad Twelves	23 × 41
Small Double Medium	24 × 36	Double Imperial	32 × 46

Larger sizes and odd sizes are sometimes made, but are *not common.*

BOOKS.

57. The terms *folio, quarto, octavo, duodecimo,* etc., indicate the number of leaves into which a sheet of paper is folded.

When a sheet is folded into	The book is called	And 1 sheet of paper makes
2 leaves	a Folio,	4 pp. (pages).
4 "	a Quarto or 4to,	8 "
8 "	an Octavo or 8vo,	16 "
12 "	a Duodecimo or 12mo,	24 "
16 "	a 16mo,	32 "
18 "	an 18mo,	36 "
24 "	a 24mo,	48 "
32 "	a 32mo,	64 "

COPYING.

58. Clerks and copyists are often paid by the *folio* for making copies of legal papers, records, and documents.

72	words make	1	folio, or sheet of	common law.
90	"	1	"	chancery.

A folio varies in different States and countries, but usually contains from 75 to 100 words.

59. ROMAN LONG MEASURES.

	Inches.		Feet.	Inches.
Digit	= .72575	Cubit	1	5.406
Uncia (inch)	= .967	Passus	4	10.02
Pes (foot)	= 11.604	Mile (millarium)	4842	

60. JEWISH LONG MEASURES.

	Feet.		Feet.
Cubit	= 1.824	Mile (4000 cubits)	= 7296
Sabbath day's journey	= 3648	Day's journey	= 33.164 mi.

61. MISCELLANEOUS.

	Feet.		Feet.
Arabian foot	= 1.095	Hebrew foot	= 1.212
Babylonian foot	= 1.140	" cubit	= 1.817
Egyptian finger	= .06145	" sacred cubit	= 2.002

RAILROAD FREIGHT.

62. When convenient to weigh them, all goods are billed at *actual weight;* but ordinarily, the articles named below are billed, at the rates given in the following

TABLE.

Ale or Beer,	820	lbs.	per bbl.	Highwines,	350	lbs.	per bbl.
Apples, green,	150	“	“	Lime,	200	“	“
Beef,	320	“	“	Nails,	108	“	per keg.
Barley,	48	“	per bu.	Oil,	400	“	per bbl.
Beans,	60	“	“	Oats,	32	“	per bu.
Cider,	350	“	per bbl.	Pork,	320	“	per bbl.
Corn Meal,	220	“	“	Potatoes, com'n,	150	“	“
Corn, shelled	56	“	per bu.	Salt, fine,	300	“	“
Corn in ear,	70	“	“	“ coarse,	350	“	“
Clover Seed,	60	“	“	“ in sacks,	200	“	per sack.
Eggs,	200	“	per bbl.	Wheat,	60	“	per bu.
Fish,	300	“	“	Whiskey,	350	“	per bbl.
Flour,	200	“	“	2000 pounds are reckoned 1 *ton.*			

Generally from 18000 to 20000 pounds is considered a car load.

63. *Lumber* and some other articles are estimated as follows:

	Weight.	Amount for car load.
PINE, HEMLOCK, and POPLAR, thoroughly seasoned, per thousand feet	3000	6500
BLACK WALNUT, ASH, MAPLE, and CHERRY, per thousand feet	4000	5000
PINE, HEMLOCK, and POPLAR, green, per M.	4000	5000
BLACK WALNUT, ASH, MAPLE, and CHERRY, green, per M.	4500	4000
OAK, HICKORY, and ELM, dry, per M. . . .	4000	5000
OAK, HICKORY, and ELM, green, per M. . .	5000	4000
SHINGLES, green, per thousand	375	55 M.
LATH, per thousand	500	40 M.
BRICK, common, per car load	4 lbs. each.	5000
COAL, per car load		250 bu.
STONE, undressed, per cubic yard	4000	5 cu. yd.

64. SCRIPTURE LONG MEASURES.

		Eng. mi.	*Paces.*	*Feet.*	*Inches.*
A Palm	equals	0	0	0	3.648
A Span	"	0	0	0	10.944
A Fathom	"	0	0	7	3.552
Ezekiel's reed	"	0	0	10	11.328
An Arabian pole	"	0	0	14	7.104
A Furlong	"	0	145	4.6	.00
An Eastern mile	"	1	403	1.0	.00
A Day's Journey	"	33	172	4.0	.00

65. SCRIPTURE MEASURES OF CAPACITY.

LIQUID.				DRY.			
		gal.	*pints.*			*pecks.*	*pints.*
A Caph	=	0	.625	A Gachal	=	0	.1416
A Log	=	0	.833	A Kab	=	0	2.8333
A Kab	=	0	8.333	An Omer	=	0	5.1
A Hin	=	1	2	A Seah	=	1	1
A Seah	=	2	4	An Epah	=	3	3
A Bath or Ephah	=	7	4	A Letek	=	16	0
A Homer	=	75	5	A Homer	=	32	1

66. MONEY MENTIONED IN SCRIPTURE.

		£	*s.*	*d.*		$ *cts.*
A Talent (gold)	equal	5464	5	8	=	26592.809
A Talent (silver)	"	341	10	4	=	1662.0249
A Manch or Mina	"	5	13	10	=	27.6990
A Pound (Mina).	"	3	4	7	=	15.7151
A Shekel (gold)	"	1	16	5	=	8.8612
A Shekel (silver)	"	0	2	3	=	0.5474
A Golden Daric or Dram . . .	"	1	1	10	=	5.3127
A Piece of Silver (Stater) . . .	"	0	2	7	=	0.6285
Tribute Money (Didrachm) . .	"	0	1	3½	=	0.3142
A Bekah	"	0	1	1	=	0.2636
A Piece of Silver (Drachm) . .	"	0	0	7¾	=	0.1571
A Penny (Denarius)	"	0	0	7½	=	0.1520
A Gerah	"	0	0	1	=	0.0202
A Farthing (Assarium)	"	0	0	0⅜	=	0.0076
A Mite	"	0	0	0 3/32	=	0.0018

MEASURES OF VALUE.

67. *Money* is the measure of the *value* of things or of services, and the medium of exchange in trade.

UNITED STATES MONEY.

68. *United States Money* is the legal currency of the United States.

69. The ***Unit*** of United States Money is the *Gold Dollar.*

TABLE.

10 Mills (*m.*) = 1 Cent . . *ct.*
10 Cents = 1 Dime . . *d.*
10 Dimes = 1 Dollar . $.
10 Dollars = 1 Eagle . E.

$$1\ E. = \begin{cases} 10000\ m. \\ 1000\ ct. \\ 100\ d. \\ 10\ \$. \end{cases}$$

70. The ***Coin*** of the United States consists of *gold, silver, nickel,* and *bronze,* and as fixed by the "New Coinage Act" of 1873, is as follows:

71. *Gold.* The double-eagle, eagle, half-eagle, quarter-eagle, three-dollar, and one-dollar pieces.

72. *Silver.* The *Trade*-dollar, half-dollar, quarter-dollar, the twenty-cent, and the ten-cent pieces.

73. *Nickel.* The five-cent, and three-cent pieces.

74. *Bronze.* The one-cent piece.

1. The *old* silver dollar, half-dime and three-cent pieces, the *bronze two-cent,* and the nickel one-cent pieces are no longer coined.

2. The *Trade-dollar* is designed solely for purposes of commerce and not for currency. The weight is 420 grains.

3. The *Standard Purity* of the *gold* and *silver* coins is .9 pure metal, and .1 alloy. The alloy of *gold* coins is *silver* and *copper;* the silver, by law, not to exceed $\frac{1}{10}$ of the whole alloy. The alloy of *silver* coins is *pure copper.*

4. The *five-cent* and *three-cent* pieces are composed of $\frac{3}{4}$ copper and $\frac{1}{4}$ nickel. The *cent* is composed of 95 parts of copper and 5 parts of tin and zinc.

CANADA MONEY.

75. ***Canada Money*** is the legal currency of the *Dominion* of Canada. The denominations are *dollars, cents,* and *mills,* and have the same *nominal* value as the corresponding denominations of U. S. Money.

The Currency of the Dominion of Canada was made *uniform* July 1st, 1871. Previous to 1858 sterling money was in use.

76. The ***Coin*** of the Dominion of Canada is *silver* and *bronze.*

77. The ***Silver Coins*** are the fifty-cent, twenty-five-cent, ten-cent, and five-cent pieces.

78. The ***Bronze Coin*** is the one-cent piece.

The *standard* silver coins consist of 925 parts (.925) pure silver and 75 parts (.075) copper. That is, they are .925 fine.

1. The *gold coin* used in Canada is the British *Sovereign,* worth 4.86\frac{2}{3}$, and the *Half-Sovereign.*

2. The intrinsic value of the 50-cent piece in United States money is about 46$\frac{1}{5}$ cents, of the 25-cent piece 23$\frac{1}{10}$ cents. In ordinary business *transactions,* they pass the same as U. States coin.

ENGLISH MONEY.

79. *English* or ***Sterling Money*** is the lega currency of Great Britain.

80. The ***Unit*** of English Money is the *Sovereign,* or *Pound Sterling.*

The value of a *Sovereign* in United States Money is $4.8665.

TABLE.

				U. S. Value.
4 Farthings (*far.*)	=	1 Penny	*d.*	£1 = 950 *far.*
12 Pence	=	1 Shilling . . .	*s.*	240 *d.*
20 Shillings	=	1 Sovereign, or .	*sov.*	20 *s.*
		1 Pound	£.	

OTHER DENOMINATIONS.

				U. S. Value.
2 Shillings (*s.*)	=	1 Florin . . .	*fl.*	$.48665.
5 Shillings	=	1 Crown . . .	*cr.*	.2166+.

81. The ***Coin*** of Great Britain in *general use* consists of *gold, silver,* and *copper,* as follows :

82. *Gold.* The sovereign, and half-sovereign.

83. *Silver.* The crown, half-crown, florin, shilling, six-penny, and three-penny piece.

84. *Copper.* The penny, half-penny, and farthing.

The *standard* gold coin contains 11 parts pure gold and 1 part alloy; silver coin 37 parts pure silver and 3 *parts* alloy.

FRENCH MONEY.

85. *French Money* is the legal currency of France, and is *decimal.*

The Franc of the REPUBLIC.

86. The ***Unit*** of French Money is the *Silver Franc.*

The Franc of the EMPIRE.

The value of a *Franc* in United States Money is $.193.

TABLE.

10 Millimes (*m.*) = 1 Centime . . . *ct.*
10 Centimes = 1 Decime . . . *dc.*
10 Decimes = 1 Franc . . . . *fr.*
20 Francs = 1 Napoleon . . *Nap.*

$$1\ Nap. = \begin{cases} 2000\ m. \\ 2000\ ct. \\ 200\ dc. \\ 20\ fr. \end{cases}$$

87. The ***Coin*** of France consists of *gold, silver,* and *bronze,* as follows :

88. *Gold.* The 100, 40, 20, 10, and 5 franc pieces.

89. *Silver.* The 5, 2, and 1 franc, the 50 and the 25 centime pieces.

90. *Bronze.* The 10, 5, 2 and 1 centime pieces.

The *standard* gold and silver coins contain 9 parts of *pure metal* and 1 part of *alloy.*

The U. S. Congress, by the Act of 1873, fixed the weight of the silver *half-dollar* at 12½ metrical grammes, so that 2 *half-dollars* are precisely *equivalent* in value to the 5 *franc* silver coin of Europe.

GERMAN MONEY.

91. The ***New Empire of Germany*** has adopted a new and uniform system of coinage.

92. The ***Unit*** of this new German System of Coinage is the *Reichsmark.*

The value of a Reichsmark ("*Mark*") in U. S. Money is $.238.

A pound of gold .900 fine is divided into 139½ pieces, and the $\frac{1}{10}$ part of this gold coin is called a "Mark," and this is subdivided into 100 pennies (*Pfennige*).

The Coin of the New Empire consists of *gold, silver* and *nickel.*

Gold. The 20, 10, and 5 mark pieces.

Silver. The 2, and 1 mark, and the 20-penny pieces.

Nickel. The 10, and the 5-penny, and pieces of less valuation.

The 10-mark piece (*gold*) is equal to 3⅓ P. Thalers (old).
The 1-mark (*silver*) is equal to 10 S. Groschen, or 1000 pennies.
The 20-penny (*silver*) is equal to 2 S. Groschen, or ⅕ of a mark.
The 10-penny (*nickel*) is equal to 1 S. Groschen, or $\frac{1}{10}$ of a mark.

JAPAN MONEY.

93. ***Japan*** has a new and *decimal* system of coinage.

94. The ***Unit*** of Japan money is the gold *Yen,* valued at $.997 U. S. money.

The ***Coin*** of Japan embraces *five gold* coins, valued at $20, $10, $5, $2, and $1. Also *five silver* coins, valued at 5, 10, 20, 50, and *100 cents,* respectively.

The weight of the new trade dollar is 420 gr., and 9 pure silver.

96. *The following shows the manner in which quotations of foreign exchange are made in this country, and as quoted Jan. 2, 1875:*

		Sixty Days.	*Three Days.*
London	Prime Bankers' Sterl. Bills	4.85½ @ 4.86	4.90 @ 4.90½
Do.	Good do. do.	4.85 @ 4.85½	4.89½ @ 4.90
Do.	Prime Commercial do.	4.84 @ 4.85	4.88½ @ 4.89½
Paris	Francs	5.17½ @ 5.16¼	5.13¾ @ 5.12½
Antwerp	Francs	5.17½ @ 5.16¼	5.13¾ @ 5.12½
Switzerland	Francs	5.17½ @ 5.16¼	5.13¾ @ 5.12½
Amsterdam	Guilders	41⅛ @ 41¼	41½ @ 41⅝
Hamburg	Reichsmarks	94⅝ @ 95⅛	96 @ 96¼
Frankfort	Reichsmarks	94⅞ @ 95⅛	96 @ 96¼
Bremen	Reichsmarks	94⅞ @ 95⅛	96 @ 96¼
Berlin	Reichsmarks	94⅞ @ 95⅛	96 @ 96¼

In the above, "Prime Bankers' Bills" are those on the most reliable banking houses; "Good" is applied to those of somewhat inferior credit; and "Prime Commercial" are merchants' drafts, which usually command a less price in the market. The quotations in the *first column* are those of 60-day bills, and in the second column those of 3 days.

97. *Rates of Exchange at London, and on London.*

EXCHANGE AT LONDON, JAN. 2, 1875.			EXCHANGE ON LONDON.		
ON	TIME.	RATE.	DATE.	TIME.	RATE.
Amsterdam	short.	11.13½@11.16½	Jan. 2.	short.	11.82
Antwerp.		25.47½@25 52¼	"		25.17
Hamburg		20.78 @20.82	"	3 mo.	20.25
Paris.	short.	25.15 @25.25	"	short.	25.19
Paris	3 months.	25.47½@25.52½			
Vienna		11.37½@11.42	Jan. 2.	3 mo.	110.50
Berlin		20.78 @20.82	"	short.	6.24¾
Frankfort		20.78 @20.82	"		119½
St. Petersburg.		32½@ 32⅝	"		
Cadiz		48⅛@ 48⅜			
Lisbon	90 days.	52⅜@ 52⅝			
Milan	3 months.	28.17½@28.22½			
Genoa		28.17½@28.22½			
Naples		28.17½@28.22½			
New York			Dec. 31.	60 days.	$4 86
Rio de Janeiro.			Dec. 17	90 days.	26⅜ @ 26½
Buenos Ayres.					
Valparaiso					
Bombay			Dec. 31	6 mo.	1s. 10⅝d.
Calcutta.			Dec. 29.		1s. 10 11/16d.
Hong Kong			Dec. 24.		4s. 2¼d.
Shanghai			Dec. 25.		5s. 8¾d. @ 5s. 9d.
Alexandria			Dec. 30.	3 mo.	96¾

98. *Weight, Fineness, and Value of Foreign* GOLD *Coins, as determined by United States Mint Assays.*

Country.	Denomination.	Weight.	Fineness.	Value in U.S. *gold* coin.		
		Ounces.	*Thous'dths.*	$	*cts.*	*m.*
Austria	Fourfold ducat	0.448	986	9	13	2
Do.	Souverain (no longer coined)	0.363	900	6	75	4
Do.	4 florins	0.104	900	1	93	5
Belgium	25 francs	0.254	899	4	72	0
Brazil	20 milreis	0.575	916,5	10	89	4
Centr'l America	2 escudos	0.209	853,5	3	68	8
Do. do.	4 reals	0.027	875	0	48	8
Chili	10 pesos (dollars)	0.492	898	9	13	6
Colombia and S. A. generally	Old doubloon	0.867	870	15	50	3
Denmark	Old 10 thaler	0.427	895	7	90	0
Egypt	Bedidlik (100 piasters)	0.275	875	4	97	4
England	Pound or Sovereign (new)	0,256,8	916,5	4	86	5
Do.	Pound average (worn)	0,256,3	916,5	4	85	6
France	20 franc (no new issues)	0,207	899	3	84	7
Germany	Old 10 thaler (Prussian)	0.427	903	7	97	1
Greece	20 drachms	0.185	900	3	44	2
India (British)	Mohur, or 15 rupees	0.375	916,5	7	10	5
Italy	20 lire (francs)	0.207	899	3	84	7
Japan	Cobang (obsolete)	0.289	572	3	57	6
Do.	New 20 yen	1.072	900	19	94	4
Mexico	Old doubloon (average)	0.867	870	15	59	3
Do.	20 pesos (empire)	1.086	875	19	64	3
Do.	20 pesos (republic), new	1.081	873	19	51	5
Netherlands	10 guilders	0.215	899	3	99	7
New Granada	10 pesos (dollars)	0.525	891,5	9	67	5
Peru	20 soles	1.055	898	19	21	3
Portugal	Coroa (crown)	0.308	912	5	80	7
Russia	5 roubles	0.210	916	3	97	6
Spain	100 reales	0.268	896	4	96	4
Do.	80 reales	0.215	869,5	3	86	4
Do.	10 escudos	0.270,8	896	5	01	5
Sweden	Ducat	0.111	975	2	23	7
Do.	Carolin (10 francs)	0.104	900	1	93	5
Tunis	25 piasters	0.161	900	2	99	5
Turkey	100 piasters	0.231	915	4	37	0

1. Foreign *gold* coins, if converted into United States coins, are subject to a charge of *one-fifth* of *one per cent.*

2. For *silver* coins there is no fixed legal valuation, as compared with gold. The value of the silver coins January 1, 1874, was computed at the rate of 120 cents per ounce, 900 fine, payable in *subsidiary silver* coin, or 113 cents in gold.

TABLE FOR INVESTORS.

99. *The following Table shows the rate per cent. of Annual Income from Bonds bearing* 5, 6, 7, *or* 8 *per cent. interest, and costing from* 40 *to* 125.

Purchase Price.	5%.	6%.	7%.	8%.	Purchase Price.	5%.	6%.	7%.	8%.
40	12.50	15.00	17.50	20.00	83	6.02	7.22	8.43	9.63
41	12.20	14.64	17.08	19.52	84	5.95	7.14	8.33	9.52
42	11.90	14.28	16.66	19.04	85	5.88	7.05	8.23	9.41
43	11.63	13.95	16.28	18.61	86	5.81	6.97	8.13	9.30
44	11.36	13.63	15.90	18.18	87	5.74	6.89	8.04	9.19
45	11.11	13.32	15.56	17.78	88	5.68	6.81	7.94	9.09
46	10.86	13.04	15.21	17.39	89	5.61	6.74	7.86	8.98
47	10.63	12.77	14.90	17.02	90	5.55	6.66	7.77	8.88
48	10.41	12.50	14.53	16.66	91	5.49	6.59	7.69	8.79
49	10.20	12.25	14.29	16.33	92	5.43	6.52	7.60	8.69
50	10.00	12.00	14.00	16.00	93	5.37	6.45	7.52	8.60
51	9.80	11.76	13.72	15.68	94	5.31	6.38	7.44	8.51
52	9.61	11.53	13.46	15.38	95	5.26	6.31	7.36	8.42
53	9.43	11.32	13.20	15.09	96	5.20	6.25	7.29	8.33
54	9.25	11.11	12.96	14.81	97	5.15	6.18	7.21	8.24
55	9.00	10.90	12.72	14.54	98	5.10	6.12	7.14	8.16
56	8.92	10.70	12.50	14.28	99	5.05	6.06	7.07	8.08
57	8.77	10.52	12.27	14.03	100	5.00	6.00	7.00	8.00
58	8.62	10.34	12.06	13.79	101	4.95	5.94	6.93	7.92
59	8.47	10.16	11.86	13.55	102	4.90	5.88	6.86	7.84
60	8.33	10.00	11.66	13.33	103	4.85	5.82	6.79	7.76
61	8.19	9.83	11.47	13.11	104	4.80	5.76	6.72	7.69
62	8.06	9.67	11.29	12.90	105	4.76	5.71	6.66	7.61
63	7.93	9.52	11.11	12.69	106	4.71	5.66	6.60	7.54
64	7.81	9.37	10.93	12.50	107	4.67	5.60	6.54	7.47
65	7.69	9.23	10.76	12.30	108	4.62	5.55	6.48	7.40
66	7.57	9.09	10.60	12.12	109	4.58	5.50	6.42	7.33
67	7.46	8.95	10.44	11.94	110	4.54	5.45	6.36	7.27
68	7.35	8.82	10.29	11.76	111	4.50	5.40	6.30	7.20
69	7.24	8.69	10.14	11.59	112	4.46	5.35	6.25	7.14
70	7.14	8.57	10.00	11.43	113	4.42	5.30	6.19	7.07
71	7.04	8.45	9.85	11.26	114	4.38	5.26	6.14	7.01
72	6.94	8.33	9.72	11.11	115	4.35	5.21	6.08	6.95
73	6.84	8.21	9.58	10.95	116	4.31	5.17	6.03	6.89
74	6.75	8.10	9.45	10.80	117	4.27	5.12	5.98	6.83
75	6.66	8.00	9.33	10.66	118	4.23	5.08	5.93	6.77
76	6.57	7.89	9.21	10.52	119	4.20	5.04	5.88	6.72
77	6.49	7.79	9.00	10.38	120	4.16	5.00	5.83	6.66
78	6.41	7.69	8.97	10.25	121	4.13	4.95	5.78	6.61
79	6.32	7.59	8.86	10.12	122	4.09	4.91	5.73	6.55
80	6.25	7.50	8.75	10.00	123	4.06	4.87	5.69	6.50
81	6.17	7.40	8.64	9.87	124	4.03	4.83	5.65	6.45
82	6.09	7.31	8.53	9.75	125	4.00	4.80	5.60	6.40

98. *Weight, Fineness, and Value of Foreign* GOLD *Coi* *as determined by United States Mint Assays.*

Country.	Denomination.	Weight.	Fineness.	Value in *gold coi*		
		Ounces.	*Thous'dths.*	$	*cts.*	
Austria	Fourfold ducat	0.448	986	9	13	
Do.	Souverain (no longer coined)	0.363	900	6	75	
Do.	4 florins	0.104	900	1	93	
Belgium	25 francs	0.254	899	4	72	
Brazil...........	20 milreis	0.575	916,5	10	89	
Centr'l America	2 escudos..............	0.209	853,5	3	68	
Do. do.	4 reals................	0.027	875	0	48	
Chili	10 pesos (dollars)	0.492	898	9	13	
Colombia and S. A. generally.	Old doubloon.............	0.867	870	15	50	
Denmark	Old 10 thaler.....	0.427	895	7	90	
Egypt...........	Bedidlik (100 piasters)......	0.275	875	4	97	
England	Pound or Sovereign (new)..	0,256,8	916,5	4	86	
Do.	Pound average (worn)... ..	0,256,3	916,5	4	85	[illegible]
France.........	20 franc (no new issues)	0,207	899	3	84	7
Germany	Old 10 thaler (Prussian) ...	0.427	903	7	97	1
Greece	20 drachms	0.185	900	3	44	2
India (British).	Mohur, or 15 rupees	0.375	916,5	7	10	5
Italy	20 lire (francs)	0.207	899	3	84	7
Japan.........	Cobang (obsolete)	0.289	572	3	57	6
Do.......... ..	New 20 yen....	1.072	900	19	94	4
Mexico	Old doubloon (average).....	0.867	870	15	59	3
Do.	20 pesos (empire)...	1.066	875	19	64	3
Do.	20 pesos (republic), new	1.081	873	19	51	5
Netherlands	10 guilders.	0.215	809	3	99	7
New Granada..	10 pesos (dollars).....	0.525	891,5	9	67	5
Peru	20 soles............	1.055	898	19	21	3
Portugal	Coroa (crown)...............	0.308	912	5	80	7
Russia....	5 roubles	0.210	916	3	97	6
Spain...........	100 reales,....................	0.268	896	4	96	4
Do.	80 reales	0.215	869,5	3	86	4
Do.	10 escudos.....	0.270,8	896	5	01	5
Sweden	Ducat	0.111	975	2	23	7
Do.	Carolin (10 francs).........	0.104	900	1	93	5
Tunis.........	25 piasters..................	0.161	900	2	99	5
Turkey....... .	100 piasters.......	0.231	915	4	37	0

1. Foreign *gold* coins, if converted into United States coins, are subject to a charge of *one-fifth* of *one per cent.*

2. For *silver* coins there is no fixed legal valuation, as compared with gold. The value of the silver coins January 1, 1874, was computed at the rate of 120 cents per ounce, 900 fine, payable in subsidiary silver coin, or 113 cents in gold.

TABLE FOR INVESTORS.

99. *The following Table shows the rate per cent. of Annual Income from Bonds bearing* 5, 6, 7, *or* 8 *per cent. interest, and costing from* 40 *to* 125.

Purchase Price.	5%.	6%.	7%.	8%.	Purchase Price.	5%.	6%.	7%.	8%.
40	12.50	15.00	17.50	20.00	83	6.02	7.22	8.43	9.63
41	12.20	14.64	17.08	19.52	84	5.95	7.14	8.33	9.52
42	11.90	14.28	16.66	19.04	85	5.88	7.05	8.23	9.41
43	11.63	13.95	16.28	18.61	86	5.81	6.97	8.13	9.30
44	11.36	13.63	15.90	18.18	87	5.74	6.89	8.04	9.19
45	11.11	13.32	15.56	17.78	88	5.68	6.81	7.94	9.09
46	10.86	13.04	15.21	17.39	89	5.61	6.74	7.86	8.98
47	10.63	12.77	14.90	17.02	90	5.55	6.66	7.77	8.88
48	10.41	12.50	14.53	16.66	91	5.49	6.59	7.69	8.79
49	10.20	12.25	14.29	16.33	92	5.43	6.52	7.60	8.69
50	10.00	12.00	14.00	16.00	93	5.37	6.45	7.52	8.60
51	9.80	11.76	13.72	15.68	94	5.31	6.38	7.44	8.51
52	9.61	11.53	13.46	15.38	95	5.26	6.31	7.36	8.42
53	9.43	11.32	13.20	15.09	96	5.20	6.25	7.29	8.33
54	9.25	11.11	12.96	14.81	97	5.15	6.18	7.21	8.24
55	9.09	10.90	12.72	14.54	98	5.10	6.12	7.14	8.16
56	8.92	10.70	12.50	14.28	99	5.05	6.06	7.07	8.08
57	8.77	10.52	12.27	14.03	100	5.00	6.00	7.00	8.00
58	8.62	10.34	12.06	13.79	101	4.95	5.94	6.93	7.92
59	8.47	10.16	11.86	13.55	102	4.90	5.88	6.86	7.84
60	8.33	10.00	11.66	13.33	103	4.85	5.82	6.79	7.76
61	8.19	9.83	11.47	13.11	104	4.80	5.76	6.72	7.69
62	8.06	9.67	11.29	12.90	105	4.76	5.71	6.66	7.61
63	7.93	9.52	11.11	12.69	106	4.71	5.66	6.60	7.54
64	7.81	9.37	10.93	12.50	107	4.67	5.60	6.54	7.47
65	7.69	9.23	10.76	12.30	108	4.62	5.55	6.48	7.40
66	7.57	9.09	10.60	12.12	109	4.58	5.50	6.42	7.33
67	7.46	8.95	10.44	11.94	110	4.54	5.45	6.36	7.27
68	7.35	8.82	10.29	11.76	111	4.50	5.40	6.30	7.20
69	7.24	8.69	10.14	11.59	112	4.46	5.35	6.25	7.14
70	7.14	8.57	10.00	11.43	113	4.42	5.30	6.19	7.07
71	7.04	8.45	9.85	11.26	114	4.38	5.26	6.14	7.01
72	6.94	8.33	9.72	11.11	115	4.35	5.21	6.08	6.95
73	6.84	8.21	9.58	10.95	116	4.31	5.17	6.03	6.89
74	6.75	8.10	9.45	10.80	117	4.27	5.12	5.98	6.83
75	6.66	8.00	9.33	10.66	118	4.23	5.08	5.93	6.77
76	6.57	7.89	9.21	10.52	119	4.20	5.04	5.88	6.72
77	6.49	7.79	9.00	10.38	120	4.16	5.00	5.83	6.66
78	6.41	7.69	8.97	10.25	121	4.13	4.95	5.78	6.61
79	6.32	7.59	8.86	10.12	122	4.09	4.91	5.73	6.55
80	6.25	7.50	8.75	10.00	123	4.06	4.87	5.69	6.50
81	6.17	7.40	8.64	9.87	124	4.03	4.83	5.65	6.45
82	6.09	7.31	8.53	9.75	125	4.00	4.80	5.60	6.40

CARLISLE TABLE.

103. *Showing the values of Annuities on Single Lives, according to the Carlisle Table of Mortality.*

AGE.	4 per cent.	5 per ct.	6 per ct.	7 per ct.	AGE.	4 per ct.	5 per ct.	6 per ct.	7 per ct.
0	14.28164	12.083	10.439	9.177	45	14.10460	12.648	11.428	10.397
1	16.55455	13.995	12.078	10.605	46	13.88928	12.480	11.296	10.292
2	17.72616	14.983	12.925	11.342	47	13.66208	12.301	11.154	10.178
3	18.71508	15.824	13.652	11.978	48	13.41914	12.107	10.998	10.052
4	19.23133	16.271	14.042	12.322	49	13.15312	11.892	10.823	9.908
5	19.59203	16.590	14.325	12.574	50	12.86902	11.660	10.631	9.749
6	19.74502	16.735	14.460	12.698	51	12.56581	11.410	10.422	9.573
7	19.79019	16.790	14.518	12.756	52	12.25793	11.154	10.208	9.392
8	19.76443	16.786	14.526	12.770	53	11.94503	10.892	9.988	9.205
9	19.69114	16.742	14.500	12.704	54	11.62673	10.624	9.761	9.011
10	19.58339	16.669	14.448	12.717	55	11.29961	10.347	9.524	8.807
11	19.45857	16.581	14.384	12.669	56	10.96607	10.063	9.280	8.595
12	19.33493	16.494	14.321	12.621	57	10.62559	9.771	9.027	8.375
13	19.20937	16.406	14.257	12.572	58	10.28647	9.478	8.772	8.153
14	19.08182	16.316	14.191	12.522	59	9.96331	9.199	8.529	7.940
15	18.95534	16.227	14.126	12.473	60	9.66333	8.940	8.304	7.743
16	18.83636	16.144	14.007	12.429	61	9.39809	8.712	8.108	7.572
17	18.72111	16.066	14.012	12.389	62	9.13676	8.487	7.913	7.403
18	18.60656	15.987	13.956	12.348	63	8.87150	8.258	7.714	7.229
19	18.48649	15.904	13.897	12.305	64	8.59330	8.016	7.502	7.042
20	18.36170	15.817	13.835	12.259	65	8.30719	7.765	7.281	6.847
21	18.23196	15.726	13.769	12.210	66	8.00906	7.503	7.049	6.641
22	18.09886	15.628	13.697	12.156	67	7.69980	7.227	6.803	6.421
23	17.95016	15.525	13.621	12.098	68	7.37976	6.941	6.546	6.180
24	17.80058	15.417	13.541	12.097	69	7.04881	6.643	6.277	5.945
25	17.64486	15.303	13.456	11.972	70	6.70936	6.336	5.998	5.690
26	17.48586	15.187	13.368	11.904	71	6.35773	6.015	5.704	5.420
27	17.32023	15.065	13.275	11.832	72	6.02548	5.711	5.424	5.162
28	17.15412	14.942	13.182	11.759	73	5.72465	5.435	5.170	4.927
29	16.99683	14.827	13.096	11.693	74	5.45812	5.190	4.944	4.719
30	16.85215	14.723	13.020	11.636	75	5.23901	4.989	4.760	4.549
31	16.70511	14.617	12.942	11.578	76	5.02399	4.792	5.579	4.382
32	16.55246	14.506	12.860	11.516	77	4.82473	4.609	4.410	4.227
33	16.39072	14.387	12.771	11.448	78	4.62166	4.422	4.238	4.067
34	16.21943	14.260	12.675	11.374	79	4.39345	4.210	4.040	3.883
35	16.04123	14.127	12.573	11.295	80	4.18289	4.015	3.858	3.713
36	15.85577	13.987	12.465	11.211	81	3.95309	3.790	3.656	3.523
37	15.66586	13.843	12.354	11.124	82	3.74034	3.606	3.474	3.352
38	15.47129	13.695	12.239	11.033	83	3.53409	3.406	3.286	3.174
39	15.27184	13.542	12.120	10.939	84	3.32856	3.211	3.102	2.999
40	15.07363	13.390	12.002	10.845	85	3.11515	3.009	2.900	2.815
41	14.88314	13.245	11.890	10.757	86	2.92831	2.830	2.739	2.652
42	14.69466	13.101	11.779	10.671	87	2.77593	2.685	2.599	2.519
43	14.50529	12.957	11.608	10.585	88	2.68337	2.597	2.515	2.439
44	14.30874	12.806	11.551	10.494	89	2.57704	2.495	2.417	2.344

ANNUITY TABLE.

104. *Showing the* PRESENT WORTH *of an Annuity of One Dollar per annum, at Compound Interest, from 1 year to 40, inclusive.*

Yrs.	3 per ct.	3½ per ct.	4 per ct.	5 per ct.	6 per ct.	7 per ct.
1	0.970 874	0.966 184	0.961 538	0.952 381	0.943 396	0.934 579
2	1.913 470	1.899 694	1.886 095	1.859 410	1.833 393	1.808 017
3	2.828 611	2.801 637	2.775 091	2.723 248	2.073 012	2.624 314
4	3.717 098	3.673 079	3.629 895	3.545 951	3.465 106	3.387 209
5	4.579 707	4.515 052	4.451 822	4.329 477	4.212 364	4.100 195
6	5.417 191	5.328 553	5.242 137	5.075 692	4.917 324	4.766 537
7	6.230 283	6.114 544	6.002 055	5.786 373	5.582 381	5.389 286
8	7.019 692	6.873 956	6.732 745	6.463 213	6.209 744	5.971 295
9	7.786 109	7.607 687	7.435 332	7.107 822	6.801 692	6.515 228
10	8.530 203	8.316 605	8.110 896	7.721 735	7.360 087	7.023 577
11	9.252 624	9.001 551	8.760 477	8.306 414	7.886 875	7.498 699
12	9.954 004	9.663 334	9.385 074	8.863 252	8.383 844	7.942 671
13	10.634 955	10.302 738	9.985 648	9.393 573	8.852 683	8.257 635
14	11.296 073	10.920 520	10.563 123	9.898 641	9.294 984	8.745 452
15	11.937 935	11.517 411	11.118 387	10.379 658	9.712 249	9.107 898
16	12.561 102	12.094 117	11.652 296	10.837 770	10.105 895	9.446 632
17	13.166 118	12.651 321	12.165 669	11.274 066	10.477 260	9.763 206
18	13.753 513	13.189 682	12.659 297	11 689 587	10.827 603	10.059 070
19	14.323 799	13.709 837	13.133 939	12.085 321	11.158 116	10.335 578
20	14.877 475	14.212 403	13.590 326	12.462 210	11.469 421	10.593 997
21	15.415 024	14.697 974	14.029 160	12.821 153	11.764.077	10.835 527
22	15.936 917	15.167 125	14.451 115	13.163 003	12.041 582	11.061 241
23	16.443 608	15.620 410	14.856 842	13.488 574	12.303 379	11.272 187
24	16.935 542	16.058 368	15.246 963	13.798 642	12.550 358	11.469 334
25	17.413 148	16.481 415	15.622 080	14.093 945	12.783 356	11.653 583
26	17.876 842	16.890 352	15.982 769	14.275 185	13.003 166	11.825 779
27	18.327 031	17.285 365	16.329 586	14.643 034	13.210 534	11.986 709
28	18.764 108	17.667 019	16.663 063	14.898 127	13.406 164	12.137 111
29	19.188 455	18.035 767	16.983 715	15.141 074	13.590 721	12.277 674
30	19.600 441	18.392 045	17.292 033	15.372 451	13.764 831	12.409 041
31	20.000 428	18.736 276	17.588 494	15.592 811	13.929 086	12.531 814
32	20.338 766	19.068 865	17.873 552	15.802 677	14.084 043	12.646 555
33	20.765 792	19.390 208	18.147 646	16.002 549	14.230 230	12.753 790
34	21.131 837	19.700 684	18.411 198	16.192 204	14.368 141	12.854 009
35	21.487 220	20.000 661	18.664 613	16.374 194	14.498 246	12.947 672
36	21.832 252	20.290 494	18.908 282	16.546 852	14.620 987	13.035 208
37	22.167 235	70.570 525	19.142 579	16.711 287	14.736 780	13.117 017
38	22.492 462	20.841 087	19.367 864	16.867 893	14.846 019	13.193 473
39	22.808 215	21.102 500	19.584 485	17.017 041	14.949 075	13.264 928
40	23.114 772	21.355 072	19.792 774	17.159 086	15.046 297	13.331 709

105. LATITUDE AND LONGITUDE OF PRINCIPAL PLACES ON THE GLOBE.

Compiled from the records of the U. S. Coast Survey and Topographical Engineer Corps, and the Imperial Gazetteer. Longitude reckoned from the Meridian of Greenwich.

Place.	Latitude.			Longitude.		
N. & S. AMERICA.		N.			W.	
Albany, N. Y ...	42°	39′	50″	73°	44′	49″
Ann Arb'r, Mich.	42	16	48	83	43	03
Annapolis, Md..	38	58	42	76	29	06
Augusta, Me....	44	18	43	69	50	
Austin, Texas...	30	13	30	97	39	
Baltimore, Md ..	30	17	48	76	36	39
Bangor, Me.....	44	48	20	68	45	42
Boston, Mass ...	42	21	30	71	03	30
Brooklyn Navy Yard, N. Y ...	40	42		73	58	30
Buffalo, N. Y....	42	50		78	59	
Burlington, Vt..	44	27		73	10	
		S.				
Buenos Ayres...	34	36	08	58	22	
		N.				
Cambr'ge, Mass.	42	22	48	71	07	40
Cape May, N. J.	38	55	48	74	57	18
		S.				
Cape Horn......	55	59		67	16	
		N.				
Charleston, S. C.	32	46	44	79	55	39
Chicago, Ill.....	41	53	48	87	37	47
Cincinnati, Ohio	39	05	54	84	29	31
Columbia, S. C..	33	59	57	81	01	54
Concord, N. H..	43	12	29	71	29	
Council Bluffs, Nebraska T...	41	30		95	48	
Des Moines, Io.	41	35		93	40	
Detroit, Mich...	42	19	46	83	2	23
Dover, Del......	39	10		75	30	
Dubuque, Iowa.	42	29	55	90	39	57
Fred'ksburg, Va.	38	18	6	77	27	17
Falls of St. Anthony, Minn ..	44	58	40	93	10	30
Fort Laramie, Wyoming T...	42	12	10	104	47	43
Ft. Leav'wth, Ks.	39	21	14	94	44	
Frankfort, Ky...	38	14		84	40	
Galveston, Tex..	29	18	17	94	46	59
Georgetown, Bermuda, W. I. ..	32	22	2	64	37	6
		S.				
Guayaquil	2	13		79	53	
		N.				
Havana	23	9		82	21	23
Halifax	44	39	4	63	35	
Harrisburg, Pa..	40	16		76	50	
Hartford, Conn..	41	45	59	72	40	45
Indianap'lis, Ind.	39	55		86	5	
N. & S. AMERICA.		N.			W.	
Jeffer'n City, Mo.	38°	36′		92°	8′	
Key West, Fla..	24	33		81	47	18″
		S.				
Lima............	12	3		77	6	
		N.				
Little Rock, Ark.	34	40		92	12	
Louisville, Ky..	38	3		85	30	
Mexico, Mexico.	19	25	45″	99	5	6
Milwaukie, Wis.	43	2	24	87	54	4
Mobile, Ala	30	41	26	88	1	29
Montreal, C. E..	45	31		73	32	56
N. Haven, Conn.	41	18	26	72	55	25
New Orleans, La.	29	57	46	90	2	30
New York, N. Y.	40	42	43	74		3
Ottawa, C. W...	45	23		75	42	4
Philadelphia, Pa.	39	56	53	75	8	42
Petersburg, Va..	37	13	47	77	23	55
Portland, Me....	43	39	28	70	14	58
Providence, R. I.	41	49	26	71	23	59
Quebec, C. E....	46	40	12	71	12	15
Richmond, Va ..	37	32	16	77	25	43
Rochester, N. Y.	43	8	17	77	51	
		S.				
Rio Janeiro.....	22	56		43	9	
		N.				
Savannah, Ga...	32	4	52	81	5	15
Sacramento, Cal.	38	34	41	121	27	44
St. August'e, Fla.	29	48	30	81	5	
St. Louis, Mo. ..	38	37	28	90	15	16
St. Paul, Minn..	44	52	46	95	4	54
Salt Lake City..	40	46	8	112	6	8
San Francisco...	37	47	36	122	46	48
Santa Fé, N. Mex.	35	41	6	106	1	22
Springfield, Ill..	39	48		89	33	
St. Joseph's, Mo.	23	3	13	109	40	44
Syracuse, N. Y..	43	3		76	9	16
Toronto, C. W..	43	30	35	79	23	21
Trenton, N. J...	40	13	10	74	45	30
Troy, N. Y......	42	43	44	73	40	41
Utica, N. Y.....	43	6	49	75	13	
		S.				
Valparaiso......	33	2		71	41	
		N.				
WASHINGTON ...	38	53	20	77	0	15
West Point, N.Y.	41	23	26	73	57	1
Wheeling, W. Va.	40	7		80	42	
Wilmington, Del.	34	14	3	77	56	47
Worcester Mass.	42	16	17	71	48	13
Yorktown, Va..	37	13		76	34	

LATITUDE AND LONGITUDE—Continued.

Place.	Latitude.			Longitude.		
EUROPE, ASIA, AFRICA, AND THE OCEANS.		N.			E.	
Antwerp	51°	13′		4°	24′	
Alexandria	31	12		29	53	
Archangel	64	32		40	33	
Athens	37	58		23	44	
Aleppo	36	11		37	10	
Algiers	36	47		3	4	
Amsterdam	52	22		4	53	
Borneo	5			115		
Botany Bay	34	2		151	13	
Barcelona	41	23		2	11	
Bombay	18	56		72	54	
Bremen	53	5		8	49	
Berlin	52	30	16″	13	23	45″
Brussels	50	51	10	4	22	
					W.	
Cape Clear	51	26		9	29	
					E.	
Calais	50	53		1	51	
Constantinople	41	1		28	59	
Canton	23	7		113	14	
Cronstadt	59	59		29	47	
Copenhagen	55	41		12	34	
		S.				
Cape of G. Hope	33	56	3	18	28	45
		N.			W.	
Cadiz	36	32		6	18	
					E.	
Calcutta	2	34		88	20	
Corinth	37	54		22	52	
Cairo	30	3		31	18	
Ceylon	9	49		80	23	
					W.	
Dublin	53	23	12	6	20	30
					E.	
Dover	51	8		1	19	
					W.	
Edinburgh	55	57		3	12	
		S.				
Falkland Islands	15	55		5	45	
					E.	
Feejee Group	17	41		178	53	
		N.				
Florence	43	46		11	16	
GREENWICH	51	28	38	....	..	
Geneva	46	11	59	6	9	15
					W.	
Glasgow	55	52		4	16	
Gibraltar	36	7		5	22	
					E.	
Genoa	44	24		8	53	
Honolulu	21	19		157	52	
		N.				
Hamburg	53	33		9	58	
Havre	49	29			6	
EUROPE, ASIA, AFRICA, AND THE OCEANS.		N.			W.	
Jerusalem	31°	48′		37	20	
					W.	
Liverpool	53	24	47″	3		
					E.	
Leghorn	43	32		10	18	
Leipsic	51	20	20	12	22	
					W.	
Lisbon	38	42		9	9	
					E.	
Moscow	55	40		35	33	
Malta	35	54		14	30	
Messina	38	12		15	35	
Mocha	13	20		43	12	
Muscat	23	37		58	35	
Marseilles	43	18		5	22	
Manilla	14	36		121	2	
Madras	14	4	9	80	15	45″
					W.	
Madrid	40	25		3	42	
Malaga	36	43		4	26	
		S.			E.	
New Zealand	34	24		173	1	
New Hebrides	15	28		167	7	
		N.				
Niphon	34	36		138	50	35
Naples	40	50		14	16	
Odessa	46	28		30	44	
Pekin	30	54		116	28	
Palermo	38	8		13	22	
Paris	48	50	13	2	20	
Rome	41	54		12	27	
Rotterdam	51	54		4	29	
Smyrna	38	26		27	7	
Singapore	1	17		103	50	
Siam	14	55		100		
					W.	
Santa Cruz	28	28		16	16	
					S.	
Sierra Leone	8	30		13	18	
					E.	
St. Helena	15	55		32	34	
		N.				
Suez	29	59		5	45	
Stockholm	59	20	31	18	6	
St. Petersburgh	59	56		30	19	
Toulon	43	07		5	22	
Tripoli	34	54		13	11	
Tunis	36	47		10	6	
Tangier	35	47		5	54	
Venice	40	50		14	26	
Vienna	48	13		16	23	
Warsaw	52	13	5	21	2	9
		S.				
Zanzibar	6	28		39	33	

106. DIFFERENCE IN TIME BETWEEN N. Y. AND THE FOLLOWING PLACES.

F. *representing fast, and* S. *slow.*

Place.	New York.		
	hr.	*min.*	*sec.*
Albany, N. Y		1	1 S.
Augusta, Ga		41	31 "
Austin, Texas	1	31	36 "
Baltimore, Md		10	27 "
Bangor, Me		20	57 F.
Boston, Mass.		11	46 "
Berlin	5	49	35 "
Beaufort, N. C		10	38 S.
Buffalo, N. Y		19	56 "
Bombay	9	47	36 F.
Bremen	5	31	16 "
Buenos Ayres	1	2	32 "
Calcutta	10	49	20 "
Canton	12	28	56 "
Cairo, Egypt	7	1	12 "
Cape Horn		36	56 "
Chicago, Ill		54	31 S.
Cincinnati, Ohio.		41	58 "
Charleston, S. C		23	43 "
Cleveland, Ohio		31	24 "
Concord, N. H		10	4 F.
Cape of Good Hope	6	9	55 "
Constantinople	6	51	56 "
Copenhagen	5	46	16 "
Detroit, Mich		36	10 S.
Dubuque, Iowa	1	6	40 "
Dublin	4	30	38 F.
Edinburgh	4	42	12 "
Florence	5	41	4 "
Fort Leavenworth, Kans.	1	22	56 S.
Frankfort, Ky		42	40 "
Galveston, Texas	1	23	8 "
Genoa	5	31	32 F.
Geneva	5	20	37 "
Gibraltar	4	34	32 "
Glasgow	4	38	56 "
GREENWICH	4	56	"
Halifax, N. B		41	40 "
Harrisburg, Pa		11	20 S.
Havana, W. I		33	25 "
Hamburg	5	35	52 F.
Havre	4	55	36 "
Indianapolis, Ind		48	20 S.
Jackson, Miss	1		32 "
Jeddo	14	16	F.
Jerusalem	7	25	20 "
Jefferson City, Mo	1	12	32 S.
Lima		12	24 "
Lisbon	4	19	24 F.
Liverpool	4	44	"
Little Rock, Ark	1	12	48 S.
Louisville, Ky		46	"
Macon, Ga		38	30 "
Madrid	4	41	52 F.
Marseilles	5	17	28 "
Mexico	1	40	20 S.
Milwaukie, Wis		55	36 "
Mobile, Ala		56	6 "
Montreal, C. W		1	49 F.
Moscow	7	18	12 "
Naples	5	53	4 "
Nashville, Tenn		51	16 S.
New Orleans, La	1	4	10 "
New Haven, Conn		4	18 F.
Ottawa, C. W		6	48 S.
Paris	5	5	20 F.
Pekin	12	41	52 "
Palermo	5	49	28 "
Philadelphia, Pa		4	34 S.
Prairie du Chien, Wis	1	8	34 "
Providence, R. I		10	24 F.
Quebec, C. E		11	11 "
Rio Janeiro	2	3	24 "
Rome	5	45	48 "
Richmond, Va		13	43 S.
Rochester, N. Y		15	24 "
Salt Lake City, U. T.	2	32	24 "
San Francisco, Cal	3	13	47 "
Santa Fé	2	8	5 "
Sydney	5	9	32 "
Suez	7	6	16 F.
St. Petersburg	6	57	16 "
St. Helena	4	33	"
St. Louis, Mo	1	5	1 S.
St. Paul, Minn	1	16	20 "
Tallahassee, Fla		42	24 "
Toronto, C. W		21	33 "
Venice	5	53	4 F.
Vienna	6	1	32 "
WASHINGTON, D. C		12	1 S.
Wheeling, W. Va		26	48 "

SPECIFIC GRAVITIES.—WATER 1.

107. *A Table showing the weight of each substance compared with an equal volume of pure water. A cubic foot of rain-water weighs 1000 ounces, or 62½ lb. Avoir. To find the weight of a* CUBIC FOOT *of any substance named in the table, remove the decimal point three places toward the right, which is multiplying by 1000, and the result will show the number of ounces in a cubic foot.*

Substances.	Specific Grav.	Substances.	Specific Grav.
Acid, acetic	1.008	Lead, cast	11.850
" nitric	1.271	" white	7.235
" sulphuric	1.841 to 2 125	" ore	7.250
Air	.001227	Lignum vitæ	1.833
Alcohol, of commerce	.835	Lime	.804
" pure	.794	" stone	2.886
Alder wood	.800	Mahogany	1.063
Ale	1.035	Manganese	8.700
Alum	1.724	Maple	.750
Aluminum	2 560	Marble	2.716
Amber	1.064	Men (living)	.891
Amethyst	2.750	Mercury, pure	14.000
Ammonia	.875	Mica	2.750
Ash	.800	Milk	1.032
Blood, human	1.054	Nickel	8.279
Brass (about)	.800	Nitre	1.900
Brick	2.000	Oil, castor	.970
Butter	.942	" linseed	.940
Cedar	.457 to .561	Opal	2.114
Cherry	.715	Opium	1.337
Cider	1.018	Pearl	2.510
Coal, bituminous (about)	1.250	Pewter	7.471
" anthracite	1.500	Platinum (native)	17.000
Copper	8.788	" wire	21.041
Coral	2.540	Poplar	.383
Cork	.240	Porcelain	2.385
Diamond	3.530	Quartz	2.500
Earth (mean of the globe)	5.210	Rosin	1.100
Elm	.671	Salt	2.130
Emerald	2.678	Sand	1.500 to 1.800
Fir	.550	Silver, cast	10.474
Glass, flint	2.760	" coin	10.534
" plate	2.760	Slate	2.110
Gold, native	15.600 to 19.500	Steel	7.816
" pure, cast	19.258	Stone	2.000 to 2.700
" coin	17.647	Sulphur, fused	1.990
Granite	2.652	Tallow	.941
Gum Arabic	1.452	Tar	1.015
Gypsum	2 288	Tin	7.291
Honey	1.456	Turpentine, spirits of	.870
Ice	.930	Vinegar	1.013
Iodine	4.948	Walnut	.671
Iron	7.645	Water, distilled	1 000
" ore	4.900	" sea	1.028
Ivory	1.917	Wax	.897
Lard	.947	Zinc, cast	7.190

108. ABBREVIATIONS USED IN BUSINESS.

@	At.	Guar.	Guarantee.
% or Acc't	Account.	Gal.	Gallon.
Am't	Amount.	Hhds.	Hogsheads.
Ass'd	Assorted.	Ins.	Insurance.
Bal.	Balance.	Inst.	This month.
Bbl.	Barrel.	Invt.	Inventory.
Blk.	Black.	Int.	Interest.
B. L.	Bill of Lading.	Mdse.	Merchandise.
¢ or ct.	Cents.	Mo.	Month.
%	Per cent.	Net.	Without disc't.
Co.	Company.	No.	Number.
Cr.	Creditor.	Pay't	Payment.
Com.	Commission.	Pd.	Paid.
Cons't	Consignment.	Pk'gs	Packages.
Dft.	Draft.	Per	By.
Disc't	Discount.	Prem.	Premium.
Do.	The same.	Prox.	Next month.
Doz.	Dozen.	Ps.	Pieces.
Dr.	Debtor.	Rec'd	Received.
Ea.	Each.	Ship't	Shipment.
Exch.	Exchange.	Sund's	Sundries.
Exps.	Expenses.	S. S.	Steamship.
Fol.	Folio.	Ult.	Last month.
Fw'd	Forward.	Yd.	Yards.
Fr't	Freight.	Yr	Year.

16 doz. $\frac{4}{\$10}, \frac{5}{\$12}, \frac{7}{\$15}$ = 16 doz. 4 of which are at $10 per doz., 5 @ $12, and 7 @ $15.

9 doz., $\frac{3}{4}$ @ 5/ $\frac{6}{5}$ @4/6. 3 doz. No. 4 @ 5 shillings per doz., and 6 doz. No. 5 @ 4 shillings sixpence per doz.

8 × 10, or 8 by 10 in. 8 inches wide and 10 inches long.

MIXED PROBLEMS.

109. 1. The area of a map laid down on the scale of an inch to a mile is 4 sq. ft. 4 sq. in. What number of acres are represented? *Ans.* 371200A.

2. A merchant marks his goods with two prices, one for ready money, and the other for a credit of 6 mo. What ratio ought the two prices to bear to each other, allowing 6% per annum simple interest? *Ans.* 100 to 103.

3. A grocer buys two kinds of tea, one at $.80, and the other at $.50 a pound, and mixes them in the proportion of 5 parts of the cheaper to 7 parts of the dearer. At what price per pound must he sell the mixture, so as to realize 25% profit on the cost? *Ans.* $.843¾.

4. A circular plot of ground containing ½ an acre has a walk all around it on the inside, 2 yards wide. Find the area of the space within the walk. *Ans.* 68.849 P.

5. A butcher paid $152 for calves and sheep, paying $6 apiece for the calves, and $4 a head for the sheep; he sold ¼ of his calves and ⅖ of his sheep for $46, and in so doing lost 8% on their cost. How many of each did he purchase? *Ans.* 12 calves; 20 sheep.

6. I borrowed a sum of money at 6% annual interest, and loaned it at 8% interest semi-annual, receiving compound interest for the second half-year, and gained thereby at the end of the year $860. What was the sum borrowed? *Ans.* $39814.81.

7. If a reservoir 24 ft. 8 in. long by 12 ft. 9 in. wide is full of water, how many cubic feet must be drawn off to lower the surface 9 in.? *Ans.* 235⅞ cu. ft.

8. The sales of a publisher amount to $250000. One-fourth of the sales are made at a profit of 33⅓%, three-fifths at a profit of 20%, and the remainder at a loss of 25%. Find the cost of the stock sold. *Ans.* $214438.88.

9. If a cannon-ball 8 inches in diameter weighs 128 lb., what will be the weight of a similar ball 15 inches in diameter? *Ans.* 843¾ lb.

10. Smith owed Jones $1800, due July 18; he paid $560 before, and the remainder after the whole was due. When were the payments made, if they were 45 days apart? *Ans.* June 17 and Aug. 1.

11. The triangular gable of a certain building has a base of 45 feet, and the perpendicular height is 16 ft. What will the lumber cost required to cover both gables, at $1.87½ per hundred feet, board measure? *Ans.* $13.50.

12. The first of 6 boys can run 3 rods as soon as the second can run 2 rods; the second 5, as soon as the third 6; the third 7, as soon as the fourth 8; the fourth 9, as soon as the fifth 10; and the fifth 15, as soon as the sixth 14. How many rods will the sixth run while the first is running 135 rods? *Ans.* 128 rods.

13. A walk 12 feet wide surrounds a square garden. The area of the walk being ½ an acre, what is the area of the inclosed square? *Ans.* 4 A. 76.6 P.

14. What sum must a man save annually, commencing at 21 years of age, to be worth $40000 when he is 50 years old, his savings being invested at 6% compound interest? *Ans.* $543.18.

15. A bin 9 ft. 6 in. long, 6 ft. wide, and 4 ft. 3 in. deep, is full of wheat. What is its value at \$2.05 a bu.?
Ans. \$339.05.

16. There is a room whose length, breadth, and height are as the numbers 3, 2, 1, containing 4374 cu. ft. What are its dimensions, and what is the distance of its remotest corners? *Ans.* 27, 18, 9, and 33.67 ft. +.

17. If I buy 120 shares of stock at 74, 150 shares at 68, and 130 at 54, how many shares must I buy at 40, so that I may sell the whole at 60, and gain 20%? *Ans.* 606.

18. A wholesale dealer sells to a retailer at 10% profit, and the retailer sells to the consumer at 50% profit; what part of the price paid by the consumer is profit? *Ans.* $\frac{13}{33}$.

19. A man spent $\frac{1}{2}$ of all the money he had and \$$\frac{1}{2}$ more; then $\frac{1}{2}$ of the remainder and \$$\frac{1}{2}$ more; then $\frac{1}{2}$ of the remainder and \$$\frac{1}{2}$ more, when he had \$5 left. How much had he at first? *Ans.* \$47.

20. Four men took a prize worth \$6213, which they agree to divide in proportion to the following fractions: A, B, and C are to have $\frac{47}{60}$; B, C, and D, $\frac{37}{60}$; A, C, and D, $\frac{7}{10}$; and A, B, and D, $\frac{3}{4}$. What does each receive?
Ans. A, \$2130; B, \$1635; C, \$1308; D, \$1090.

21. A man bought a farm for \$10000, giving a 3 years' note, at 7% annual interest, and rents it at 10%, payable quarterly. He paid 2% annually, in advance, for repairs, and at the end of 2 yr. sold the same at 1% discount and took up his note. Reckoning 7% simple interest on all expenditures and income, did he make or lose by the transaction and how much? *Ans.* \$131.50 gain.

22. How many square feet of tin will cover a hemispherical dome, whose diameter is 10 feet? *Ans.* 157.08 sq. ft. +

23. What must be the length of a halter rope, one end fastened to a stake, that will allow a horse to graze on $\frac{1}{4}$ of an acre? *Ans.* 58.85 ft.

24. A man 34 years old insured his life for $6000, on the 10-payment plan. When he died, there was a net gain to his family of $4463.40. How many payments had he made? *Ans.* 5.

25. W. D. L. Robbins bought 50 shares of bank stock at 106. From the time of purchase until the end of 3 yr. 3 mo. he received a semi-annual dividend of 4%, when he sold the stock at 111. Money being worth 7% compound interest, what did he gain? *Ans.* $274.62.

26. A and B run a 100-rod race, and A wins by 2 rods; C and D run over the same course, and C wins by 6 rods; B and D run over it, and B wins by 4 rods. If A and C run, which will win, and by how much? *Ans.* C, by .085 rd.

27. If 2 acres of grass, together with what grows on 2 A. while they are grazing, keep 8 cows 3 wk., and in like manner 5 A. keep 15 cows 6 wk., how many acres will be required to keep 15 cows 2 wk.? *Ans.* 3 A.

28. Two casks, each containing 8 gallons of wine, P and M, are mixed as follows: 1 gallon is taken out of each and put back into the other. This is repeated 4 times; how much of the original contents remains in each cask? *Ans.* $5\frac{17}{64}$ gal.

29. Which is the cheaper, a suit of clothes which costs $56 and will last 18 months, or a suit that costs $35 and will last 10 months; and how much will a man save in 20 years who wears the cheaper suit, interest not being reckoned? *Ans.* The $56 suit; $$93\frac{1}{3}$ gain.

30. A resident at Rome, having a bequest of $12500 made him in New York, orders the remittance to be made to his agent in London, who remits the proceeds to Rome, reserving his commission of ½% on the draft sent. If the course of exchange on London is 4.90½ in New York, and the rate between London and Rome is 25.8 lire to the pound sterling, how much does the man realize from his bequest? *Ans.* 65422.14 lire.

31. Three men can do a piece of work in 30 days, A and B each doing the same amount of work in the same time; if either A or B are absent, the two others can do $\frac{5}{9}$ of the work in 28 days. In what time can each separately do the work? *Ans.* A or B, 74$\frac{2}{17}$ da.; C, 157½ da.

32. The annual rent of a farm is $1200, ⅓ payable in money, ¼ in wheat, and the balance in corn, the average prices being as follows; wheat $1.50, and corn $.65 a bu. What will be the rent when the average prices of wheat and corn are $1.75 and $.80 per bushel? *Ans.* $1365.38.

33. In a square lot containing 1⅗ A., how far is the center from each corner, and from the middle of each side? *Ans.* 11.31 rd., and 8 rd.

34. A cistern has 3 pipes; the first can fill it in 3 hr., the second in 4 hr., and the third can empty it in 1 hr. If the cistern is full, and these three pipes be opened in order at 3, 4, and 5 o'clock, at what time will the cistern be emptied? *Ans.* 12 min. past 7.

35. The fore-wheel of a carriage, which is 7 feet in circumference, makes 792 more revolutions than the hind-wheel in a distance of 3½ miles. Find the circumference and the diameter of the hind-wheel.

Ans. Circum., 10 ft.; diam., 3.183 ft.+.

36. If 2 bushels of barley are required to sow an acre, how many acres can be sowed with the contents of a bin 4 ft. long, 3 ft. wide, and 18 in. deep? *Ans.* 7.23 A.

37. Reduce to simplest form

$$\frac{\frac{3}{5}\text{ of }\left(\frac{6}{7}-\frac{3}{4}\right)}{\frac{7}{9}\text{ of }\left(\frac{3}{4}-\frac{5}{7}\right)} \times \left\{\frac{\frac{8}{15}-\frac{6}{13}}{\frac{5}{13}-\frac{3}{11}} - \frac{9\frac{1}{5}+3\frac{4}{9}}{5\frac{11}{14}\text{ of }7\frac{1}{9}} + \frac{7\frac{5}{8}-4\frac{2}{5}}{\frac{21}{23}-\frac{19}{21}}\right\}.$$

Ans. 902.

38. A mechanic is employed for 1 yr. 6 mo., at $80 a month, payable monthly, but his wages remain unpaid until the end of the whole term of service. What is due, allowing simple interest at 6% per annum? *Ans.* $1501.20.

39. The Pacific Insurance Company issued a policy of insurance on a Japan merchantman for $\frac{3}{4}$ of the estimated value of the ship and cargo, at $4\frac{1}{4}$%, and immediately re-insured $\frac{1}{2}$ the risk in the Atlantic Company, at 3%. During the outward voyage the ship was wrecked, and the Atlantic Company lost $2500 more than the Pacific Company. What did the owners lose? *Ans.* 75166\frac{2}{3}$.

40. Chas. H. Allen purchased goods of A. Roman & Co. to the amount of $4000, $1000 to be paid April 2, 1876, $1200 to be paid May 5th, $750 to be paid June 15th, and the balance to be paid June 30th. At what date must a note for the whole amount be drawn, payable in 3 months, that it fall due at the average date? *Ans.* Feb. 19, 1876.

41. A parlor 36 ft. by 24 ft. and 12 ft. high is to be carpeted with carpeting 32 in. wide, at $4.75 a yard, and papered with paper 18 in. wide, at $.30 a yard; the windows, covering $\frac{1}{12}$ of the walls, are to be glazed, at $.65 a square foot; and the doors, occupying $\frac{1}{18}$ of the walls, are to be painted, at $.75 a square yard. Find the whole cost. *Ans.* 680.33\frac{1}{3}$.

42. A quantity of wheat heaped up in a corner of a square room, forms a portion of a cone, of which the perpendicular height is 4 ft. 6 in., and the slant height 6 ft. 9 in. What is its value at $1.75 a bushel? *Ans.* $41.93.

43. Three partners commenced business, each with a capital of $10000. The gains and the losses were to be shared equally, and each partner was to receive interest at 6% on his capital, and to pay interest on all sums withdrawn. At the close of the year they had: Cash on hand, $5450; Merchandise, $15000; Bills Receivable, $1475; Personal debts due them, $6750. The firm owed: Bills payable, $1000; Personal debts, $840. During the year A drew out $1260.50, the interest on which, to the end of the year was $35.95; B drew $2420, on which the interest was $47.50; and C drew $2615.50, the interest on which was $53.25. What was their gain or loss, and what was each partner's capital at the end of the year?

Ans. $1467.70 net gain; $8621.73, B's capital.
$9792.78, A's capital; $8420.48, C's "

44. If a certain box of tea is sold for $.60 a pound, there will be a gain of $2; but if sold for $.44 a pound, there will be a loss of $6. Find the number of pounds of tea in the box, and its cost. *Ans.* 50 lb.; $28.

45. At what time between 11 and 12 o'clock are the hour and minute hands of a watch at right angles, and when directly opposite?

Ans. 11 hr. 10 min. $54\frac{6}{11}$ sec. } at right angles.
11 hr. 43 min. $38\frac{2}{11}$ sec. }
11 hr. 27 min. $16\frac{4}{11}$ sec., opposite.

46. Find the cost of raising the surface of $\frac{3}{8}$ of an acre, 9 inches, at $.65 per cubic yard of earth? *Ans.* $294.94.

47. A New Jersey capitalist is informed by his agent in Detroit that \$7200 is due him on the sale of some property. He instructs the agent to remit by draft payable in 60 days after sight, exchange being at ¾% premium. The agent by mistake remits a sight draft, which, when received in New Jersey, is *accepted*, and paid after the expiration of the 3 days' *grace*. If the capitalist puts this money at interest at the legal rate, will he gain or lose by the blunder of his agent, and how much?

Ans. \$4.59 loss.

48. Reduce to simplest form,

$$\left(\frac{4.\dot{6}\times 2.\dot{1}}{6.2\div 1.25}\div .0625\right)\times\left(\frac{.0\dot{6}\times 2.\dot{3}\times .\dot{3}\dot{6}}{(4.\dot{8}5714\dot{2}\times .4)\div .\dot{1}4285\dot{7}}\right)\div\frac{.8\times .\dot{8}\dot{1}}{.6}$$

Ans. .121166+.

49. A man rowed a distance of 1½ miles *down* a stream in 20 minutes, but without the aid of the current it would have taken him 30 minutes. What was the velocity of the stream per hour, and how long would it take him to return against it? *Ans.* Vel., 1½ mi.; 1 hr.

50. A manufacturer invests \$25000 as capital in a business, on which he can realize by hand labor an annual profit of 10%; but he buys a machine for \$5000, by which his profit on the remainder of his capital is raised 20%. This machine lasts 5 years. If at the end of each year, except the last, he draws out \$1500, and adds the remainder of the profits to his capital, what is the amount of his capital at the end of the five years, and how much greater is it than it would have been, had he not bought the machine?

Ans. \$62193.95 capital; \$29568.95 gain.

SEP 28 1928